1 Reverse, Converse, Inverse, and Contrapositive

1.1 Implications

Consider the implication formula $A \Rightarrow B$.

Its *reverse* is $B \Leftarrow A$.

Its *converse* is $B \Rightarrow A$.

Its *inverse* is $\neg A \Rightarrow \neg B$.

Its *contrapositive* is $\neg B \Rightarrow \neg A$.

By definition, the reverse of an implication means the same as the original implication itself. Each implication implies its contrapositive, even intuitionistically. In classical logic, an implication is logically equivalent to its contrapositive, and, moreover, its inverse is logically equivalent to its converse.

1.2 Binary Relations

The *reverse* or *converse* of a binary relation denoted with R is $R^{-1} := \{ \ (y,x) \mid (x,y) \in R \ \}$. Note that R^{-1} is an *inverse* (in the sense that $R \circ R^{-1} = {}_{\mathrm{dom}(R)}\!\upharpoonleft\!\mathrm{id}$ and $R^{-1} \circ R = {}_{\mathrm{ran}(R)}\!\upharpoonleft\!\mathrm{id}$ holds) iff R is an injective function.

In the tradition of Bertrand A. W. Russell, Willard Van O. Quine still calls R^{-1} *the converse of* R in his *Mathematical Logic*, rev. ed. (1981). As the converse of an implication is not logically equivalent to the original implication itself (as opposed to its reverse), and as $y\ R\ x$ is also called the *converse* of $x\ R\ y$, which is again not logically equivalent (as opposed to $y\ R^{-1}\ x$ being logically equivalent to $x\ R\ y$), this tradition is confusing, esp. when we take R to be the implication relation $\Rightarrow$. Moreover, in computer science one would speak of "the reverse of a list" (which generalizes pairs) and not of "the converse of a list". Thus, the name "reverse relation" should actually be preferred to "converse relation", but traditionalists will possibly see this differently.

There is no such thing as a *contrapositive of a relation*.