# Poole’s Specificity Revised

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# Specificity of Explanations in Logic Programm.

- [Poole, 1985]: David L. Poole.
  *On the Comparison of Theories:*
  *Preferring the Most Specific Explanation*.
  **IJCAI 9**, pp. 144–147. Binary Specificity Relation

- [Simari & Loui, 1992]: Guillermo R. Simari, Ronald P. Loui.
  *A Mathematical Treatment of Defeasible Reasoning and its Implementation*.
  Artificial Intelligence **53**, pp. 125–157.
  Specificity Relation is an Ordering

- [Stolzenburg &al., 2003]: Frieder Stolzenburg,
  Alejandro J. García, Carlos I. Chesñevar, Guillermo R. Simari.
  *Computing Generalized Specificity*.
  J. Applied Non-Classical Logics **13**, pp. 87–113.

# Theory $\mathfrak{T}_\Pi$ / Derivability $\vdash$

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The theory $\mathfrak{T}_{\Pi}$ is inductively defined to contain

- all instances of literals from $\Pi$, and
- all literals $L$ for which there is a conjunction $C$ of literals from $\mathfrak{T}_{\Pi}$ such that $L \Leftarrow C$ is an instance of a rule in $\Pi$.

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For $\mathfrak{L} \subseteq \mathfrak{T}_\Pi$, we also write $\Pi \vdash \mathfrak{L}$.

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  Default rules that hold in most situations

The set $\Pi := \Pi^{\mathrm{F}} \cup \Pi^{\mathrm{G}}$ is the set of *strict* rules that — contrary to the defeasible rules — are

- considered to be safe and
- not doubted in the concrete situation.

# Specificity [Quasi-] Orderings on Arguments

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- Non-Transitive Specificity Relations:

$\lesssim_{\mathrm{P1}}$ David Poole's original

$\lesssim_{\mathrm{P2}}$ Guillermo R. Simari's minor correction of $\lesssim_{\mathrm{P1}}$

$\lesssim_{\mathrm{P3}}$ Our minor correction of $\lesssim_{\mathrm{P2}}$

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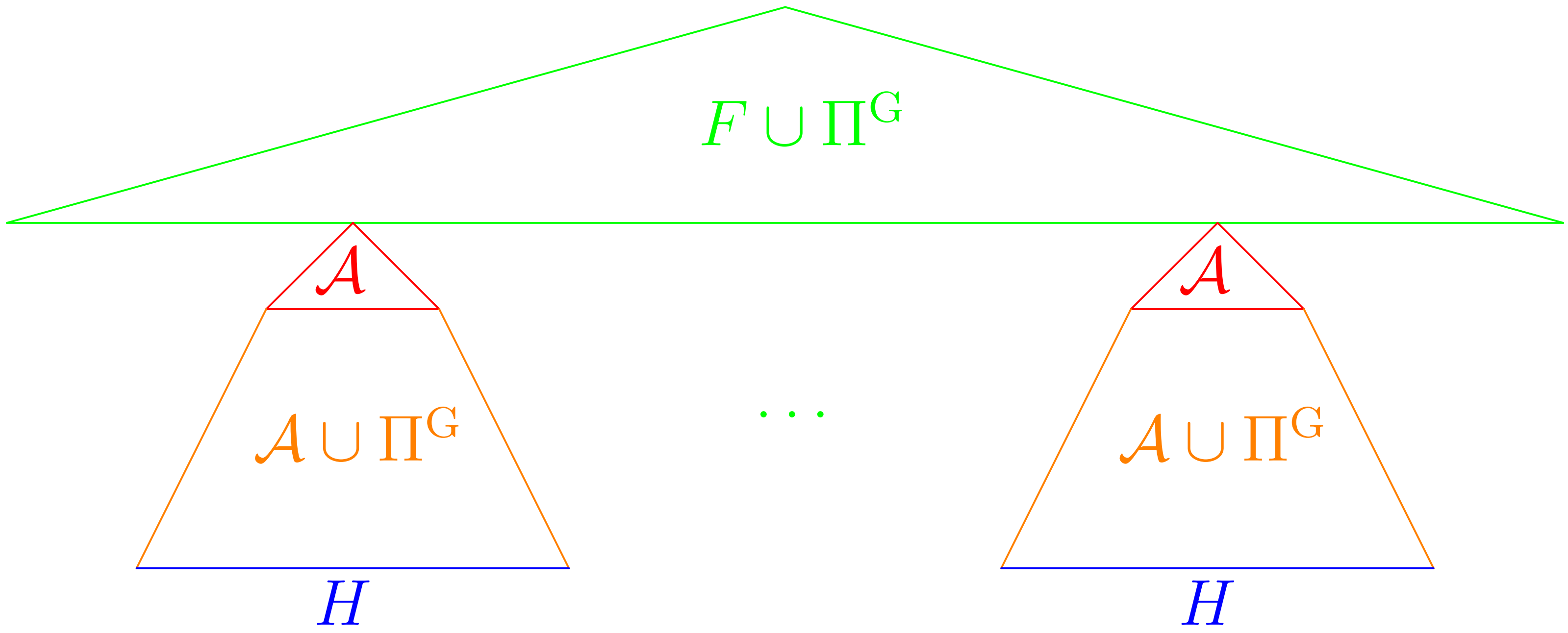
- Transitive Specificity Relations (Quasi-Orderings!):

$\lesssim_{\mathrm{CP1}}$ Compared to $\lesssim_{\mathrm{P3}}$:
– Reduction from $\mathfrak{T}_{\Pi^{\mathrm{F}} \cup \Pi^{\mathrm{G}} \cup \Delta}$ to $\mathfrak{T}_{\Pi^{\mathrm{F}} \cup \Pi^{\mathrm{G}}}$.
– Admission of $\Pi^{\mathrm{F}}$ after *completed* defeasible argumentations.

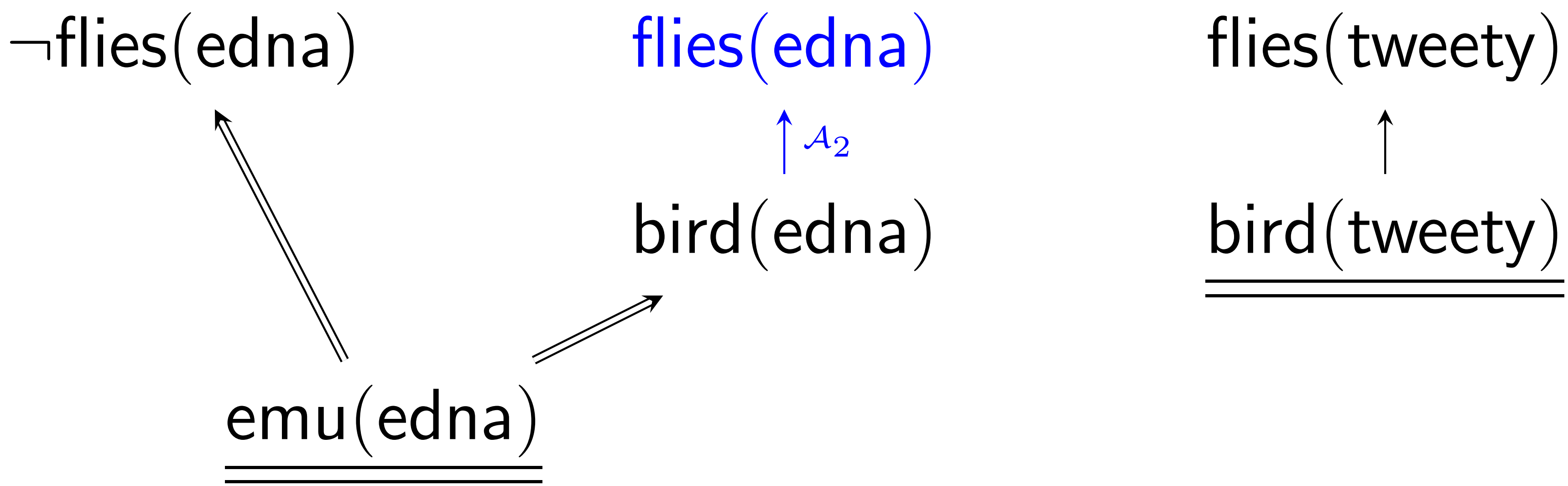
$\lesssim_{\mathrm{CP2}}$ Efficiency improvement of $\lesssim_{\mathrm{CP1}}$

# Defeasible Parts of Derivation w.r.t. $(\Pi^{\mathrm{F}}, \Pi^{\mathrm{G}}, \Delta)$



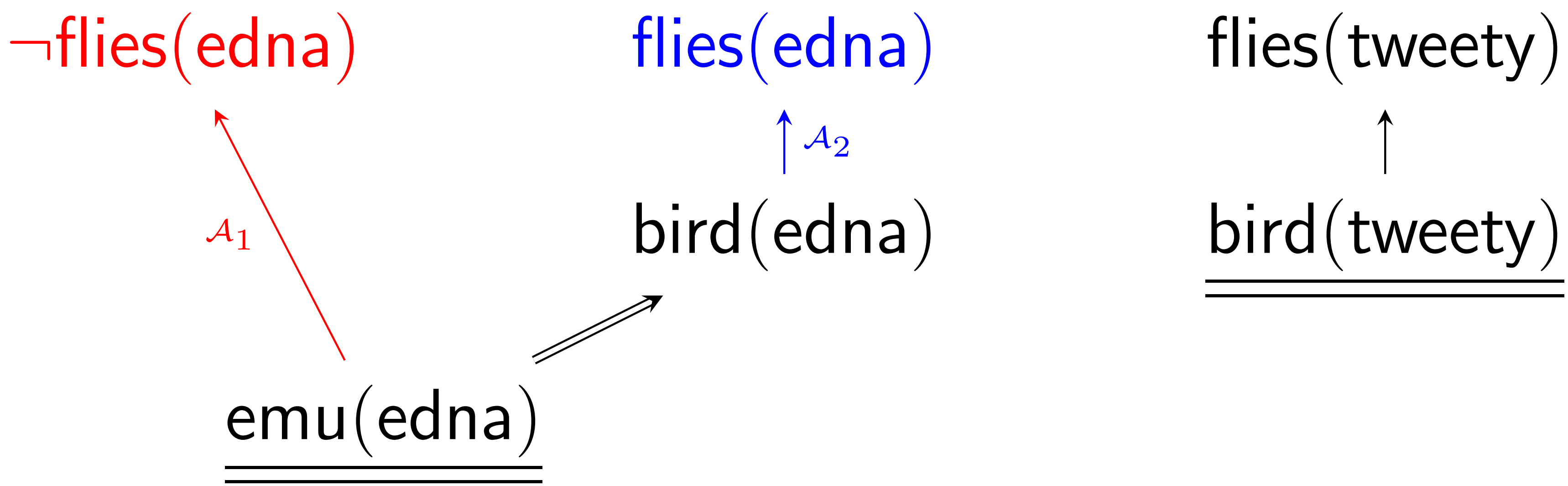

| | $H \subseteq \ldots$ | $F = \ldots$ |
|---|---|---|
| P1, P2, P3 | $\mathfrak{L}_{\Pi^{\mathrm{F}} \cup \Pi^{\mathrm{G}} \cup \Delta}$ | $H$ |
| CP1, CP2 | $\mathfrak{L}_{\Pi^{\mathrm{F}} \cup \Pi^{\mathrm{G}}}$ | $\Pi^{\mathrm{F}}$ |

# Example 1 of [Poole, 1985]




$$\Pi^{\mathrm{F}} := \{\mathsf{bird}(\mathsf{tweety}), \mathsf{emu}(\mathsf{edna})\},$$
$$\Pi^{\mathrm{G}} := \{\mathsf{bird}(x) \Leftarrow \mathsf{emu}(x),\ \ \neg\mathsf{flies}(x) \Leftarrow \mathsf{emu}(x)\},$$
$$\Delta := \{\mathsf{flies}(x) \leftarrow \mathsf{bird}(x)\}.$$
$$\mathcal{A}_2 := \{\mathsf{flies}(\mathsf{edna}) \leftarrow \mathsf{bird}(\mathsf{edna})\}.$$
$$(\emptyset, \neg\mathsf{flies}(\mathsf{edna})) <_{\mathrm{P1,P2,P3,CP1,CP2}} (\mathcal{A}_2, \mathsf{flies}(\mathsf{edna})).$$

# Ex.2 of [Poole,1985]. Pref. of "More Concise"




$\Pi^{\mathrm{F}} := \{\mathsf{bird}(\mathsf{tweety}), \mathsf{emu}(\mathsf{edna})\}, \qquad \Pi^{\mathrm{G}} := \{\mathsf{bird}(x) \Leftarrow \mathsf{emu}(x)\},$

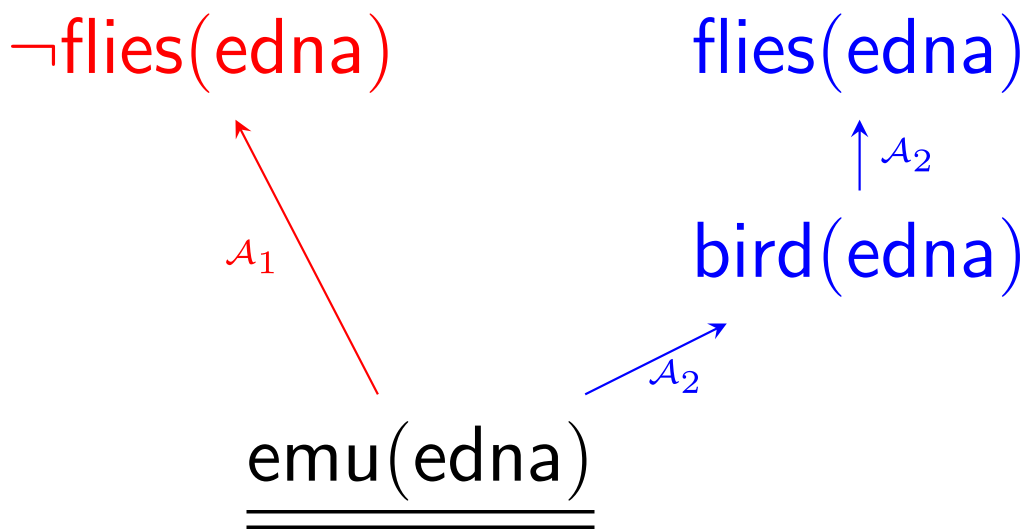
$\Delta := \{\mathsf{flies}(x) \leftarrow \mathsf{bird}(x), \ \neg\mathsf{flies}(x) \leftarrow \mathsf{emu}(x)\}.$

$\mathcal{A}_1 := \{\neg\mathsf{flies}(\mathsf{edna}) \leftarrow \mathsf{emu}(\mathsf{edna})\}.$

$\mathcal{A}_2 := \{\mathsf{flies}(\mathsf{edna}) \leftarrow \mathsf{bird}(\mathsf{edna})\}.$

$(\mathcal{A}_1, \neg\mathsf{flies}(\mathsf{edna})) <_{\mathrm{P1,P2,P3,CP1,CP2}} (\mathcal{A}_2, \mathsf{flies}(\mathsf{edna})).$

# Example 3 of [Poole, 1985] (renamed)




$\Pi^{\mathrm{F}} := \{\mathsf{emu}(\mathsf{edna})\}, \qquad \Pi^{\mathrm{G}} := \emptyset$

$\Delta := \{\mathsf{flies}(x) \leftarrow \mathsf{bird}(x),\ \ \neg\mathsf{flies}(x) \leftarrow \mathsf{emu}(x),\ \ \mathsf{bird}(x) \leftarrow \mathsf{emu}(x)\}.$

$\mathcal{A}_1 := \{\neg\mathsf{flies}(\mathsf{edna}) \leftarrow \mathsf{emu}(\mathsf{edna})\}.$

$\mathcal{A}_2 := \{\mathsf{flies}(\mathsf{edna}) \leftarrow \mathsf{bird}(\mathsf{edna}),\ \ \mathsf{bird}(\mathsf{edna}) \leftarrow \mathsf{emu}(\mathsf{edna})\}.$

$(\mathcal{A}_1, \neg\mathsf{flies}(\mathsf{edna})) \approx_{\mathrm{CP1,CP2}} (\mathcal{A}_2, \mathsf{flies}(\mathsf{edna})).$

$(\mathcal{A}_1, \neg\mathsf{flies}(\mathsf{edna})) <_{\mathrm{P1,P2,P3}} (\mathcal{A}_2, \mathsf{flies}(\mathsf{edna})).$

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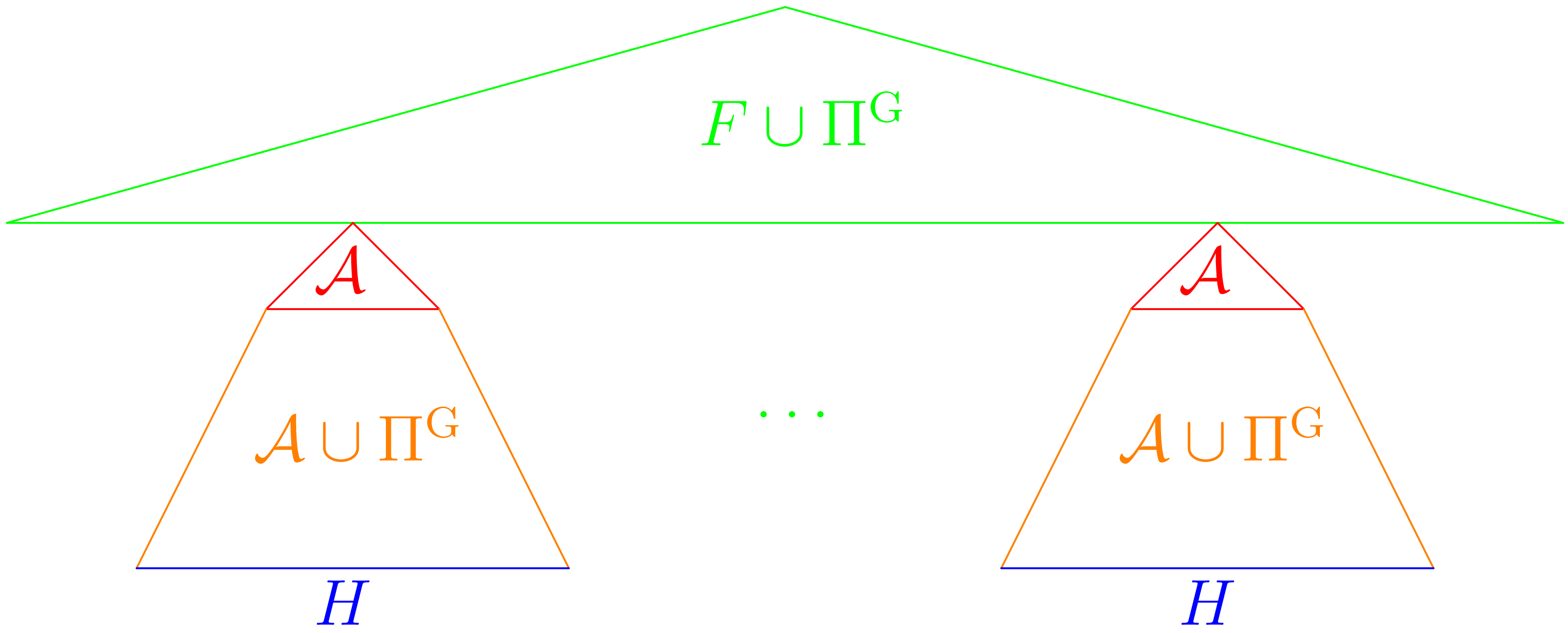


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# Computing Specificity Relations

**(phase 1)** Derive the literals that provide the basis for specificity considerations. CP1/2: $\mathfrak{T}_{\Pi^{\mathrm{F}} \cup \Pi^{\mathrm{G}}}$. P1–3: $\mathfrak{T}_{\Pi^{\mathrm{F}} \cup \Pi^{\mathrm{G}} \cup \Delta}$.

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- a subset $H$ of the literals derived in phase 1,
- the first item $\mathcal{A}$ of a given argument $(\mathcal{A}, L)$, and
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we derive a further set of literals $\mathfrak{L}$: $H \cup \mathcal{A} \cup \Pi^{\mathrm{G}} \vdash \mathfrak{L}$.

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**(phase 3)** On the basis of $\mathfrak{L}$, the literal of the argument is derived: $\mathfrak{L} \cup \Pi^{\mathrm{F}} \cup \Pi^{\mathrm{G}} \vdash \{L\}$.
P1–3: phase 3 is empty: $\mathfrak{L} = \{L\}$.
CP1/2: It is admitted to use the facts from $\Pi^{\mathrm{F}}$ in phase 3, in addition to the general rules from $\Pi^{\mathrm{G}}$.

# Definition [Minimal] [Simplified] Activation Set

Let $\mathcal{A}$ be a set of ground instances of rules from $\Delta$, and let $L$ be a literal.

$H$ is a *simplified activation set for* $(\mathcal{A}, L)$ if $L \in \mathfrak{T}_{H \cup \mathcal{A} \cup \Pi^{\mathrm{G}}}$.

$H$ is an *activation set for* $(\mathcal{A}, L)$ if, for some $\mathfrak{L} \subseteq \mathfrak{T}_{H \cup \mathcal{A} \cup \Pi^{\mathrm{G}}}$, $L \in \mathfrak{T}_{\mathfrak{L} \cup \Pi^{\mathrm{F}} \cup \Pi^{\mathrm{G}}}$.

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$H$ is a *minimal* [*simplified*] *activation set for* $(\mathcal{A}, L)$ if $H$ is an [simplified] activation set for $(\mathcal{A}, L)$, but no proper subset of $H$ is an [simplified] activation set for $(\mathcal{A}, L)$.

# Definitions of $\lesssim_{\text{CP1}}$ and $\lesssim_{\text{P3}}$

$(\mathcal{A}_1, L_1) \lesssim_{\text{CP1}} (\mathcal{A}_2, L_2)$ if $(\mathcal{A}_1, L_1)$ and $(\mathcal{A}_2, L_2)$ are arguments, and we have

1. $L_1 \in \mathfrak{L}_{\Pi^{\text{F}} \cup \Pi^{\text{G}}}$ or
2. $L_2 \notin \mathfrak{L}_{\Pi^{\text{F}} \cup \Pi^{\text{G}}}$ and
   every $H \subseteq \mathfrak{L}_{\Pi^{\text{F}} \cup \Pi^{\text{G}}}$ that is an [minimal] activation set for $(\mathcal{A}_1, L_1)$ is also an activation set for $(\mathcal{A}_2, L_2)$.

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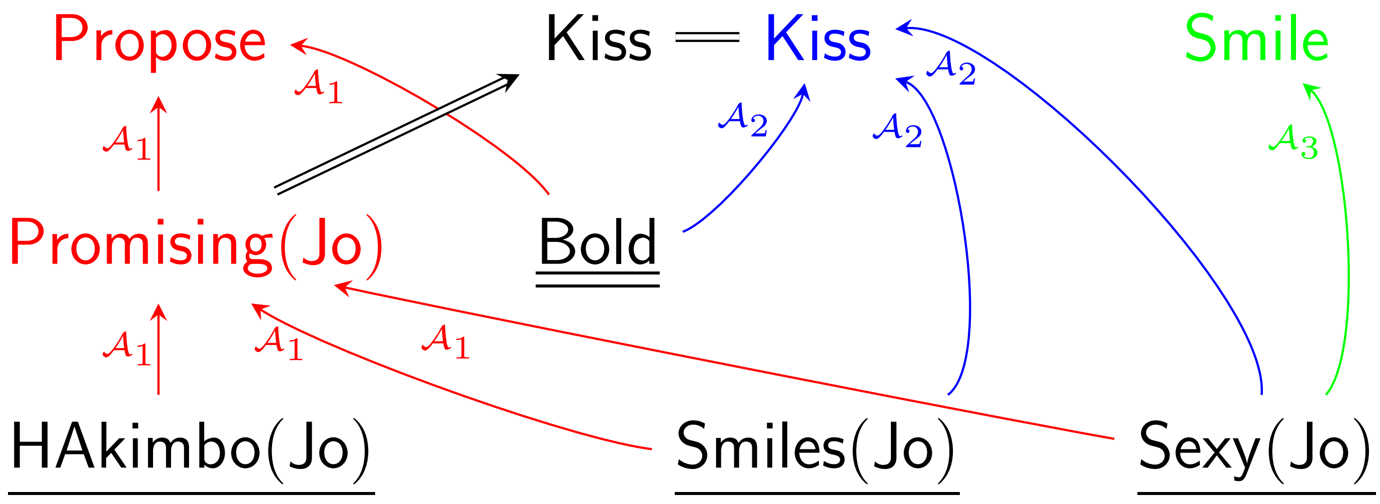
$(\mathcal{A}_1, L_1) \lesssim_{\mathrm{P3}} (\mathcal{A}_2, L_2)$ if $(\mathcal{A}_1, L_1)$ and $(\mathcal{A}_2, L_2)$ are arguments,
$L_2 \in \mathfrak{T}_{\Pi\mathrm{F}\cup\Pi\mathrm{G}}$ implies $L_1 \in \mathfrak{T}_{\Pi\mathrm{F}\cup\Pi\mathrm{G}}$, and,
for every $H \subseteq \mathfrak{T}_{\Pi\mathrm{F}\cup\Pi\mathrm{G}\cup\Delta}$
that is a [minimal] simplified activation set for $(\mathcal{A}_1, L_1)$
but not a simplified activation set for $(\emptyset, L_1)$,
$H$ is also a simplified activation set for $(\mathcal{A}_2, L_2)$.

# Example Not Transitive. Pref. of "More Precise"




$$\mathcal{A}_1 := \left\{ \begin{array}{l} \mathsf{Promising}(\mathsf{Jo}) \leftarrow \mathsf{HAkimbo}(\mathsf{Jo}) \wedge \mathsf{Smiles}(\mathsf{Jo}) \wedge \mathsf{Sexy}(\mathsf{Jo}), \\ \mathsf{Propose} \leftarrow \mathsf{Promising}(\mathsf{Jo}) \wedge \mathsf{Bold} \end{array} \right\},$$

$$\mathcal{A}_2 := \{\mathsf{Kiss} \leftarrow \mathsf{Bold} \wedge \mathsf{Smiles}(\mathsf{Jo}) \wedge \mathsf{Sexy}(\mathsf{Jo})\},$$

$$\mathcal{A}_3 := \{\mathsf{Smile} \leftarrow \mathsf{Sexy}(\mathsf{Jo})\}. \quad \Pi^{\mathrm{G}} := \{\mathsf{Kiss} \Leftarrow \mathsf{Promising}(G)\}.$$

$$\Pi^{\mathrm{F}} := \{\mathsf{Bold},\ \mathsf{HAkimbo}(\mathsf{Jo}),\ \mathsf{Smiles}(\mathsf{Jo}),\ \mathsf{Sexy}(\mathsf{Jo})\}.$$

$(\mathcal{A}_1, \mathsf{Propose}) <_{\mathrm{P1\text{--}3,CP1/2}} (\mathcal{A}_2, \mathsf{Kiss}) <_{\mathrm{P1\text{--}3,CP1/2}} (\mathcal{A}_3, \mathsf{Smile})$, but
$(\mathcal{A}_1, \mathsf{Propose}) <_{\mathrm{CP1/2}} (\mathcal{A}_3, \mathsf{Smile}) \not\gtrsim_{\mathrm{P1\text{--}3}} (\mathcal{A}_1, \mathsf{Propose})$.

# Conclusion: Novel Specificity Relations are . . .

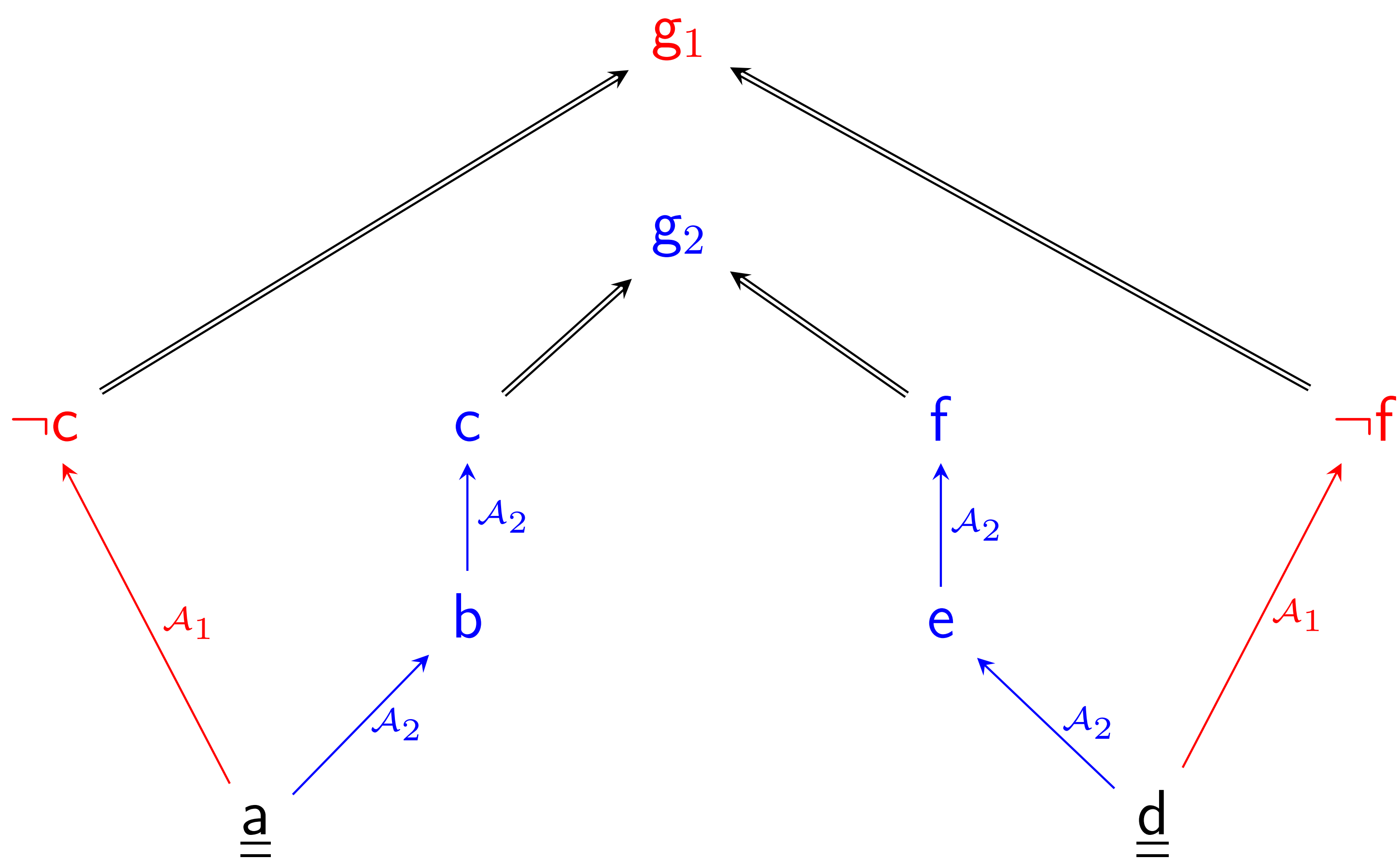
- Transitive!
- Monotonic w.r.t. Conjunction!
- Even More Intuitive!
- Slightly More Efficient!
- More Comparing?

**Theorem**: $\lesssim_{\mathrm{P3}} \subseteq \lesssim_{\mathrm{CP1}}$.

**Corollary**: $\triangle_{\mathrm{CP1}} \subseteq \triangle_{\mathrm{P3}}$.

**But in general:** $<_{\mathrm{P3}} \not\subseteq <_{\mathrm{CP1}}$. Luckily!
(Otherwise monotonicity w.r.t. $\wedge$ would be lost.)

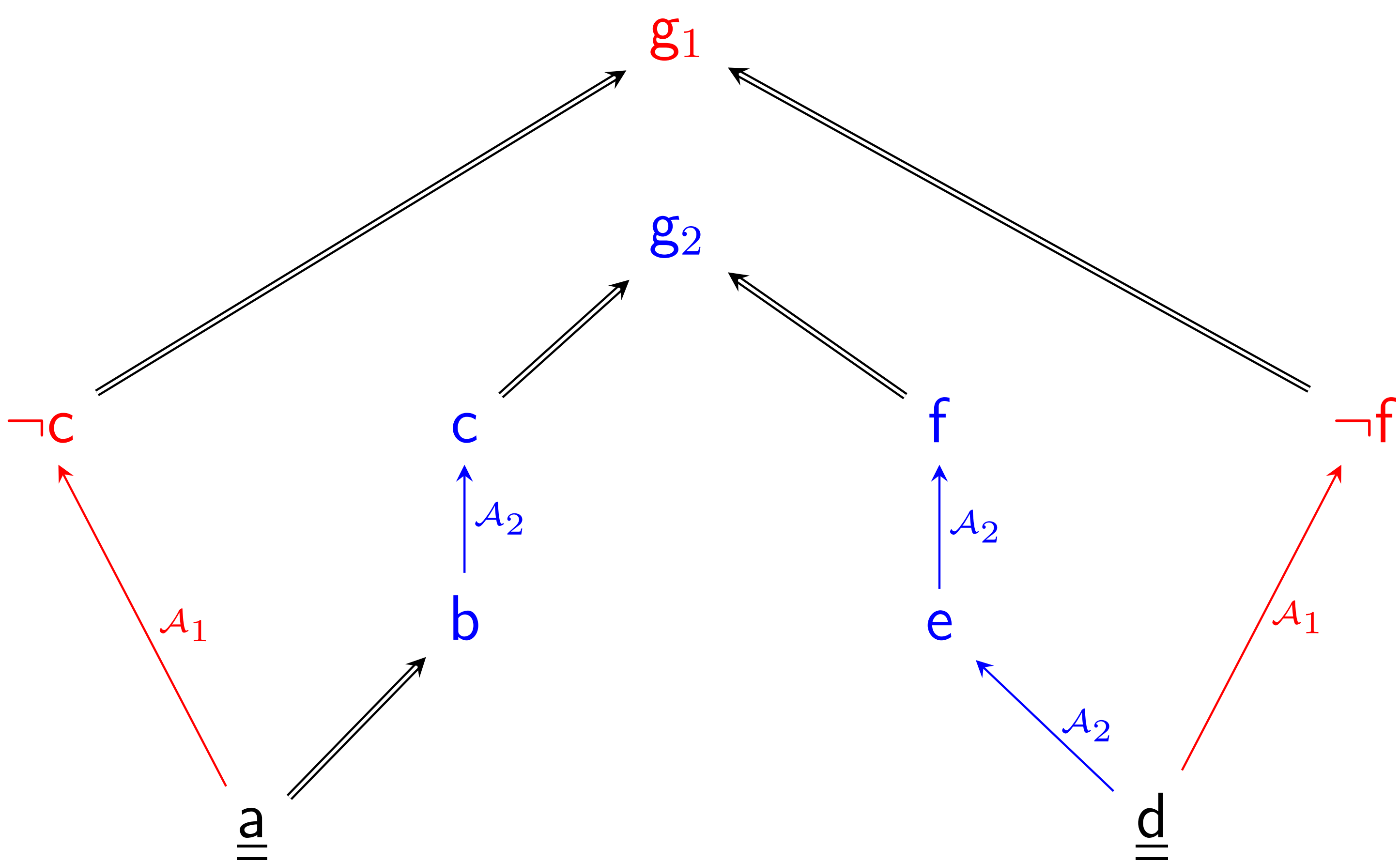
# [Poole,1985,Example 6]: Monotonicity w.r.t. $\wedge$




$(\mathcal{A}_1,\neg\mathsf{c}) <_{\mathrm{P1\text{--}3}} (\mathcal{A}_2,\mathsf{c})$, $(\mathcal{A}_1,\neg\mathsf{f}) <_{\mathrm{P1\text{--}3}} (\mathcal{A}_2,\mathsf{f})$, but $(\mathcal{A}_1,\mathsf{g}_1) \triangle_{\mathrm{P1\text{--}3}} (\mathcal{A}_2,\mathsf{g}_2)$.

$(\mathcal{A}_1,\neg\mathsf{c}) \approx_{\mathrm{CP1/2}} (\mathcal{A}_2,\mathsf{c})$, $(\mathcal{A}_1,\neg\mathsf{f}) \approx_{\mathrm{CP1/2}} (\mathcal{A}_2,\mathsf{f})$, so $(\mathcal{A}_1,\mathsf{g}_1) \approx_{\mathrm{CP1/2}} (\mathcal{A}_2,\mathsf{g}_2)$.

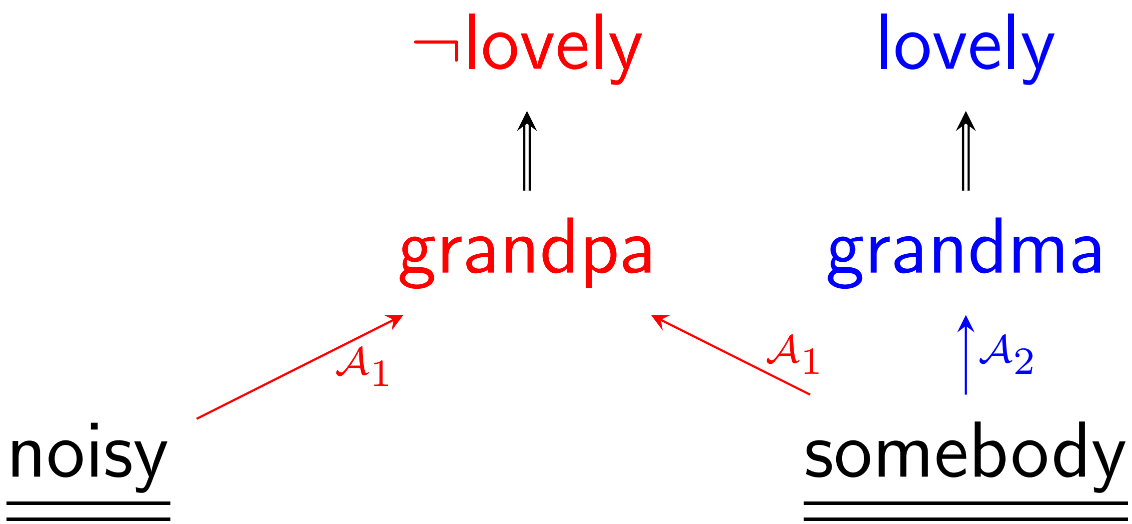
# 1st Variation of [Poole,1985,Example 6]




$(\mathcal{A}_1, \neg c) <_{\mathrm{P1\text{–}3}} (\mathcal{A}_2, c)$, $(\mathcal{A}_1, \neg f) <_{\mathrm{P1\text{–}3}} (\mathcal{A}_2, f)$, but $(\mathcal{A}_1, g_1) \triangle_{\mathrm{P1\text{–}3}} (\mathcal{A}_2, g_2)$.

$(\mathcal{A}_1, \neg c) <_{\mathrm{CP1/2}} (\mathcal{A}_2, c)$, $(\mathcal{A}_1, \neg f) \approx_{\mathrm{CP1/2}} (\mathcal{A}_2, f)$, so $(\mathcal{A}_1, g_1) <_{\mathrm{CP1/2}} (\mathcal{A}_2, g_2)$.

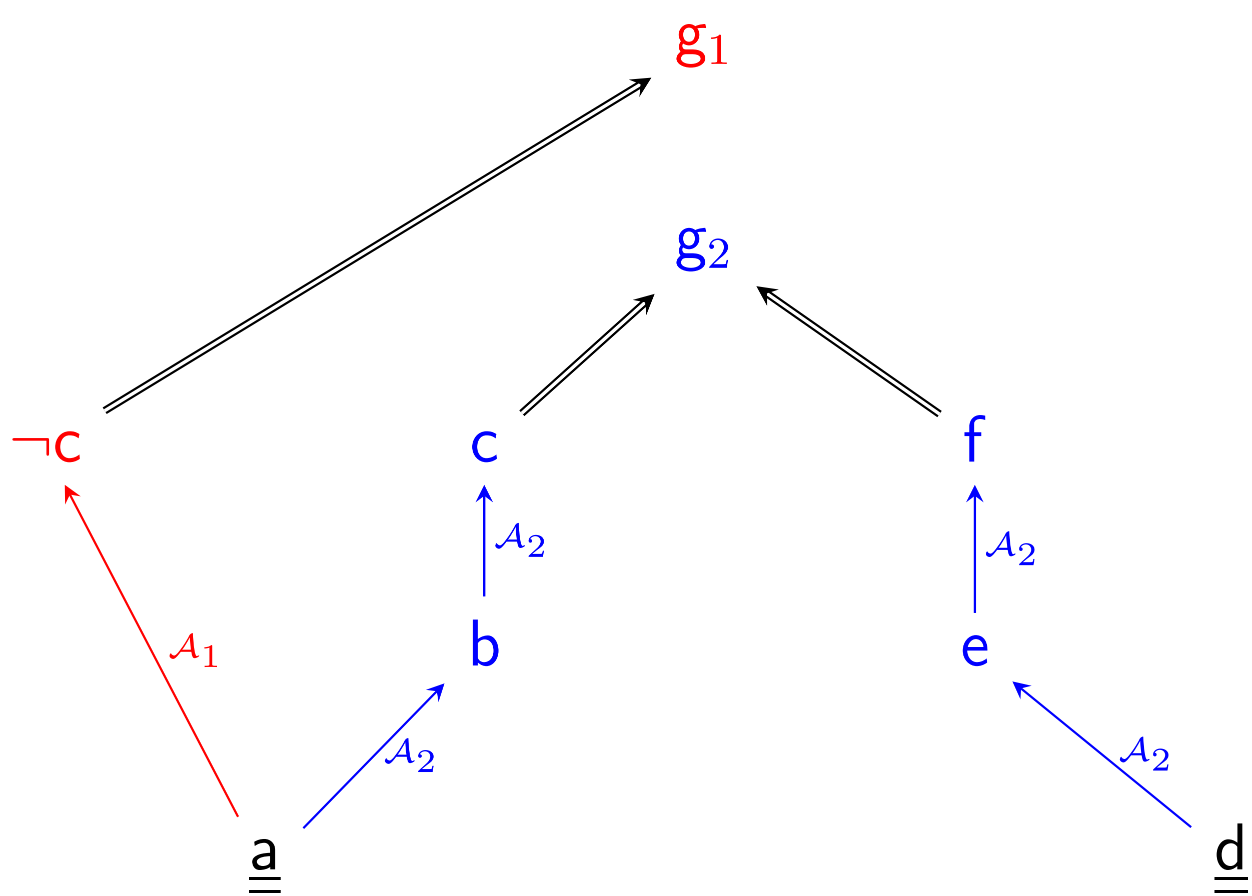
# “More Precise”




$$(\mathcal{A}_1, \neg\text{lovely}) <_{\mathrm{P1\text{–}3}} (\mathcal{A}_2, \text{lovely}).$$

$$(\mathcal{A}_1, \neg\text{lovely}) <_{\mathrm{CP1/2}} (\mathcal{A}_2, \text{lovely}).$$

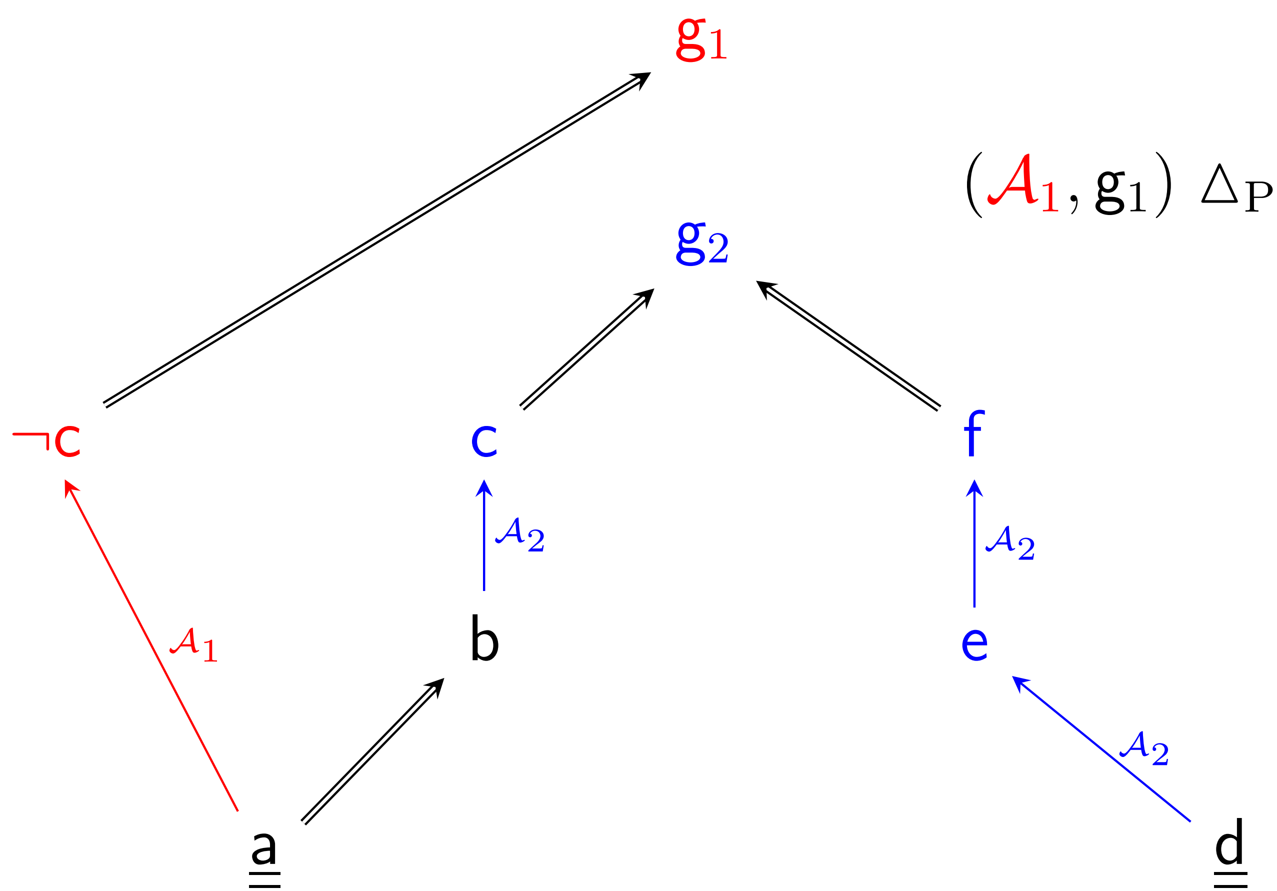
# “More Precise”, 2nd Var. of [Poole,1985,Ex. 6]




$(\mathcal{A}_1,\neg c) <_{\mathrm{P1\text{--}3}} (\mathcal{A}_2,c)$, but $(\mathcal{A}_1,g_1)\,\Delta_{\mathrm{P1\text{--}3}}\,(\mathcal{A}_2,g_2)$.

$(\mathcal{A}_1,\neg c) \approx_{\mathrm{CP1/2}} (\mathcal{A}_2,c)$, but $(\mathcal{A}_1,g_1) >_{\mathrm{CP1/2}} (\mathcal{A}_2,g_2)$, “more precise”.

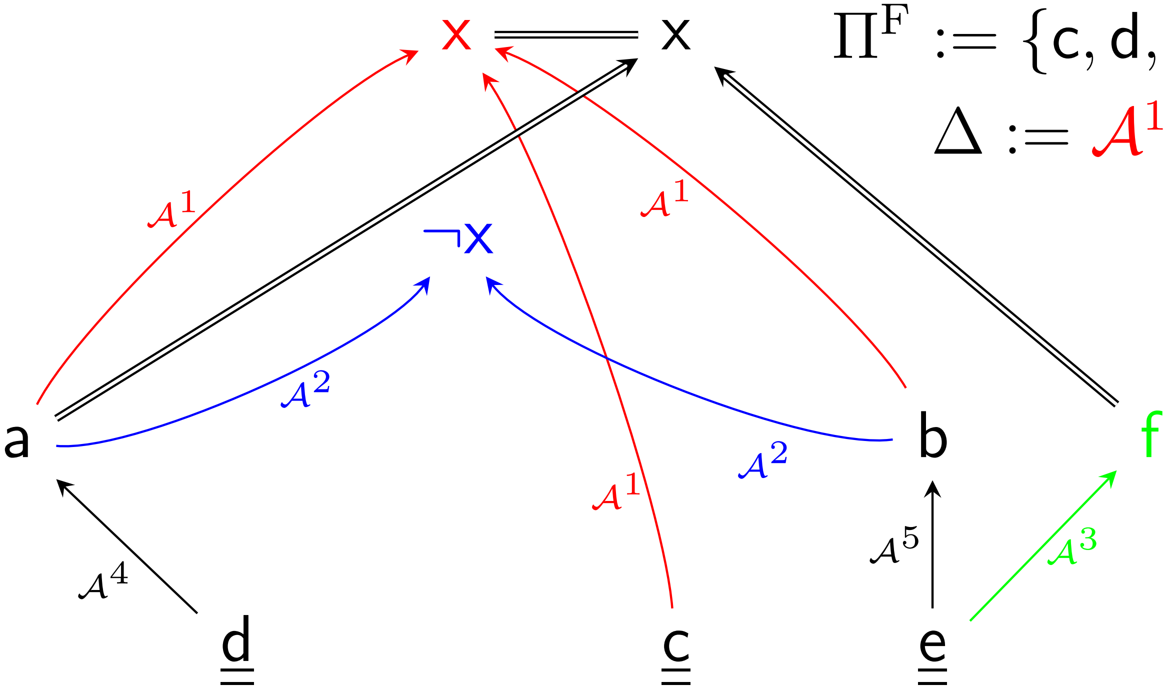
# “Precise vs. Concise”, 3rd Var. [Poole, 1985, Ex. 6]




$(\mathcal{A}_1, g_1) \vartriangle_{P1-3,CP1/2} (\mathcal{A}_2, g_2)$

The conflict between a being “more concise” than b and b∧d being “more precise” than a is indeed irresolvable.

# [Stolzenbg, 2003, Ex.11]: No Pruning for $\lesssim_{\rm P3}$!




$\Pi^{\rm F} := \{\mathsf{c}, \mathsf{d}, \mathsf{e}\}, \ \ \Pi^{\rm G} := \{\mathsf{x} \Leftarrow \mathsf{a}\wedge\mathsf{f}\},$

$\Delta := \mathcal{A}^1 \cup \mathcal{A}^2 \cup \mathcal{A}^3 \cup \mathcal{A}^4 \cup \mathcal{A}^5.$

$\mathcal{A}^1 := \{\mathsf{x} \leftarrow \mathsf{a}\wedge\mathsf{b}\wedge\mathsf{c}\}.$

$\mathcal{A}^2 := \{\neg\mathsf{x} \leftarrow \mathsf{a}\wedge\mathsf{b}\}.$

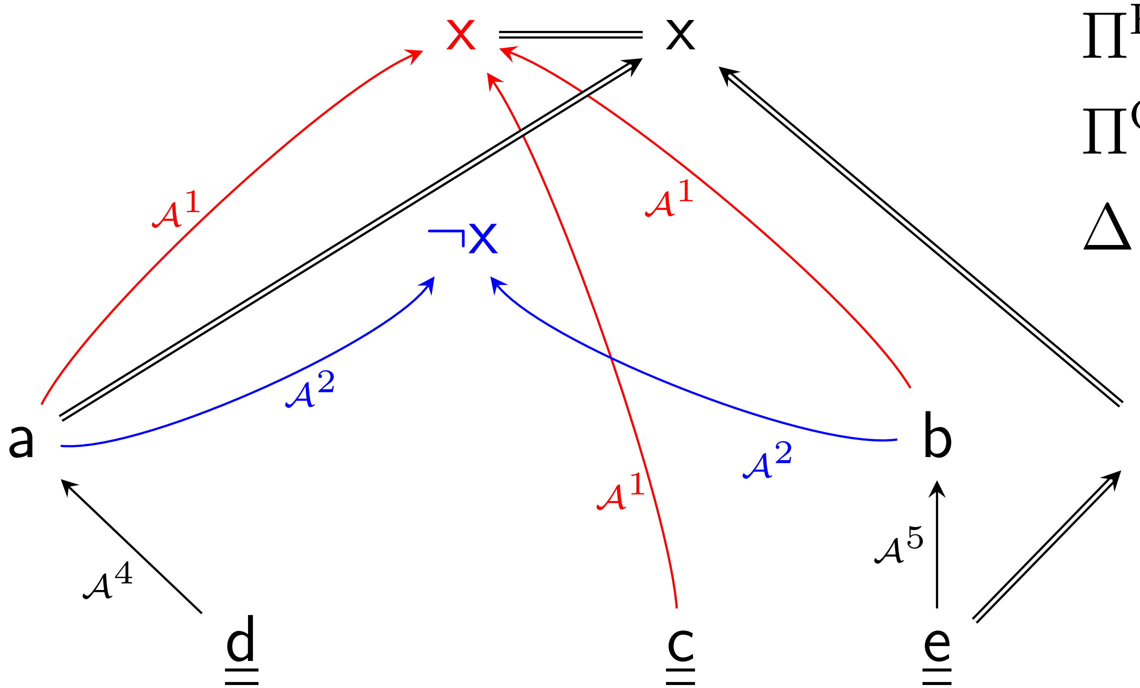
$\mathcal{A}^3 := \{\mathsf{f} \leftarrow \mathsf{e}\}.$

$\mathcal{A}^4 := \{\mathsf{a} \leftarrow \mathsf{d}\}.$

$\mathcal{A}^5 := \{\mathsf{b} \leftarrow \mathsf{e}\}.$

$(\mathcal{A}^1\cup\mathcal{A}^4\cup\mathcal{A}^5, \mathsf{x}) <_{\rm CP1/2} (\mathcal{A}^2\cup\mathcal{A}^4\cup\mathcal{A}^5, \neg\mathsf{x}) \approx_{\rm CP1/2} (\mathcal{A}^3\cup\mathcal{A}^4, \mathsf{x})$. All $\lesssim_{\rm P1-3}$-incomparable! For $(\mathcal{A}^1\cup\mathcal{A}^4\cup\mathcal{A}^5, \mathsf{x}) \not\lesssim_{\rm P3} (\mathcal{A}^2\cup\mathcal{A}^4\cup\mathcal{A}^5, \neg\mathsf{x})$ we have to consider (implicitly via $\{\mathsf{d}, \mathsf{f}\} \subseteq \mathfrak{T}_{\Pi^{\rm F}\cup\Pi^{\rm G}\cup\Delta}$) the defeasible rule of $\mathcal{A}^3$, which is not part of any of the two arguments under comparison. No pruning possible for $\lesssim_{\rm P1-3}$.

# Variation of [Stolzenbg, 2003, Ex.11]

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$\Delta := \mathcal{A}^1 \cup \mathcal{A}^2 \cup \mathcal{A}^4 \cup \mathcal{A}^5$.

$\mathcal{A}^1 := \{\mathsf{x} \leftarrow \mathsf{a} \wedge \mathsf{b} \wedge \mathsf{c}\}$.

$\mathcal{A}^2 := \{\neg\mathsf{x} \leftarrow \mathsf{a} \wedge \mathsf{b}\}$.

$\mathcal{A}^4 := \{\mathsf{a} \leftarrow \mathsf{d}\}$.

$\mathcal{A}^5 := \{\mathsf{b} \leftarrow \mathsf{e}\}$.

$(\mathcal{A}^4, \mathsf{x}) \approx_{\mathrm{CP1/2}} (\mathcal{A}^1 \cup \mathcal{A}^4 \cup \mathcal{A}^5, \mathsf{x}) >_{\mathrm{CP1/2}} (\mathcal{A}^2 \cup \mathcal{A}^4 \cup \mathcal{A}^5, \neg\mathsf{x})$.

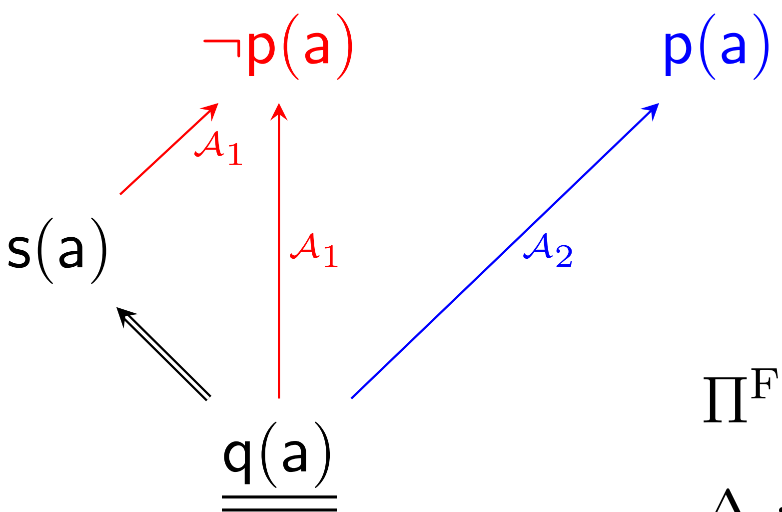
Makes sense because $\mathsf{d} \wedge \mathsf{e}$ is more precise (specific) than $\mathsf{d}$.

$\mathsf{c} \wedge \mathsf{d} \wedge \mathsf{e}$ is irrelevant because approach is model-theoretic.

$(\mathcal{A}^4, \mathsf{x}) <_{\mathrm{P1-3}} (\mathcal{A}^1 \cup \mathcal{A}^4 \cup \mathcal{A}^5, \mathsf{x}) \vartriangle_{\mathrm{P1-3}} (\mathcal{A}^2 \cup \mathcal{A}^4 \cup \mathcal{A}^5, \neg\mathsf{x}) \vartriangle_{\mathrm{P1-3}}$

(Bullshit!) $(\mathcal{A}^4, \mathsf{x})$.

# [Stolzenburg, 2003, Ex. p.95]: Global Effect!




$$\Pi^{\mathrm{F}} := \{\mathsf{q}(\mathsf{a})\},\ \ \Pi^{\mathrm{G}} := \{\mathsf{s}(x) \Leftarrow \mathsf{q}(x)\},$$

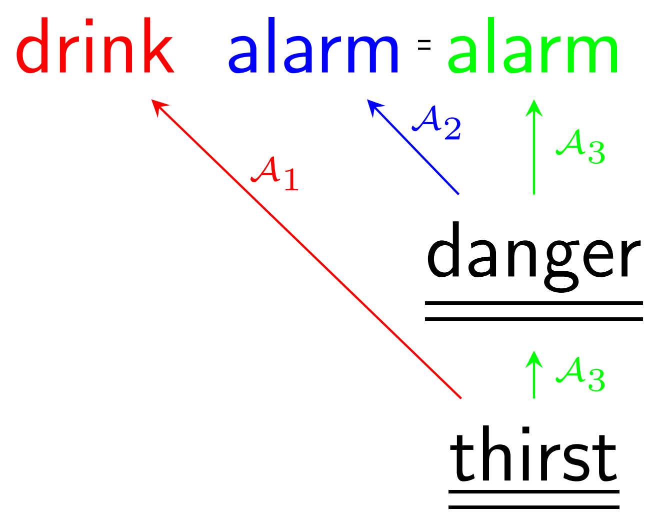
$$\Delta := \{\mathsf{p}(x) \leftarrow \mathsf{q}(x), \neg\mathsf{p}(x) \leftarrow \mathsf{q}(x) \wedge \mathsf{s}(x)\}.$$

$$\mathcal{A}_1 := \{\neg\mathsf{p}(\mathsf{a}) \leftarrow \mathsf{q}(\mathsf{a}) \wedge \mathsf{s}(\mathsf{a})\}.$$

$$\mathcal{A}_2 := \{\mathsf{p}(\mathsf{a}) \leftarrow \mathsf{q}(\mathsf{a})\}.$$

$$(\mathcal{A}_1, \neg\mathsf{p}(\mathsf{a})) \approx_{\mathrm{P1-3,CP1/2}} (\mathcal{A}_2, \mathsf{p}(\mathsf{a})).$$

# $\lesssim_{\mathrm{CP1}}$ vs. $\lesssim_{\mathrm{CP2}}$




$$\Pi^{\mathrm{F}} := \{\mathsf{thirst}, \mathsf{danger}\},$$

$$\Pi^{\mathrm{G}} := \emptyset, \quad \Delta := \mathcal{A}_1 \cup \mathcal{A}_3,$$

$$\mathcal{A}_1 := \{\mathsf{drink} \leftarrow \mathsf{thirst}\},$$

$$\mathcal{A}_2 := \{\mathsf{alarm} \leftarrow \mathsf{danger}\}.$$

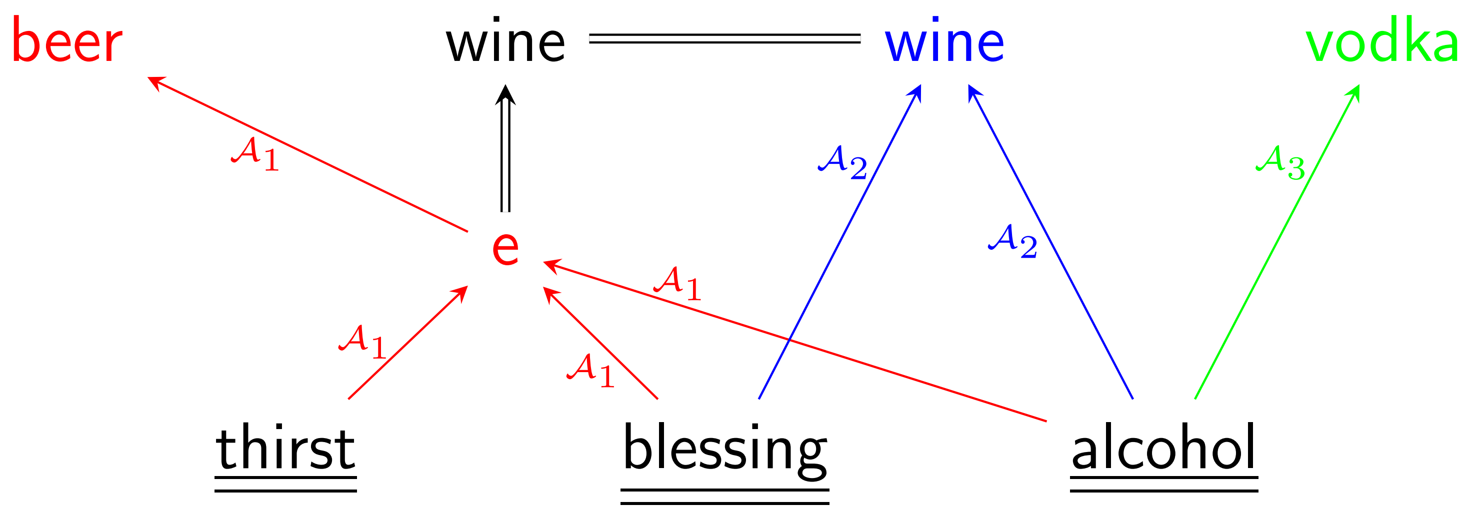
$$\mathcal{A}_3 := \mathcal{A}_2 \cup \{\mathsf{danger} \leftarrow \mathsf{thirst}\}.$$

$$(\mathcal{A}_2, \mathsf{alarm}) <_{\mathrm{CP1}} (\mathcal{A}_3, \mathsf{alarm}) \approx_{\mathrm{CP2}} (\mathcal{A}_2, \mathsf{alarm})$$

$$(\mathcal{A}_1, \mathsf{drink}) <_{\mathrm{CP1}} (\mathcal{A}_3, \mathsf{alarm}) \,\Delta_{\mathrm{CP2}}\, (\mathcal{A}_1, \mathsf{drink})$$

$$(\mathcal{A}_1, \mathsf{drink}) \,\Delta_{\mathrm{CP1}}\, (\mathcal{A}_2, \mathsf{alarm}) \,\Delta_{\mathrm{CP2}}\, (\mathcal{A}_1, \mathsf{drink})$$

# Example Not Transitive. Pref. of “More Precise”




$\mathcal{A}_1 := \{\text{e} \leftarrow \text{alcohol}\wedge\text{blessing}\wedge\text{thirst},\ \ \text{beer} \leftarrow \text{e}\}$,

$\mathcal{A}_2 := \{\text{wine} \leftarrow \text{alcohol}\wedge\text{blessing}\}$,

$\mathcal{A}_3 := \{\text{vodka} \leftarrow \text{alcohol}\}$, $\quad \Delta := \mathcal{A}_1 \cup \mathcal{A}_2 \cup \mathcal{A}_3$,

$\Pi^{\mathrm{F}} := \{\text{alcohol}, \text{blessing}, \text{thirst}\}$, $\quad \Pi^{\mathrm{G}} := \{\text{wine} \Leftarrow \text{e}\}$,

$(\mathcal{A}_1, \text{beer}) <_{\mathrm{P1\text{--}3,CP1/2}} (\mathcal{A}_2, \text{wine}) <_{\mathrm{P1\text{--}3,CP1/2}} (\mathcal{A}_3, \text{vodka})$, but

$(\mathcal{A}_1, \text{beer}) <_{\mathrm{CP1/2}} (\mathcal{A}_3, \text{vodka}) \not\gtrsim_{\mathrm{P1\text{--}3}} (\mathcal{A}_1, \text{beer})$.

# All you need is Quasi-Ordering

A *quasi-ordering* is a reflexive transitive relation.
An *(irreflexive) ordering* is an irreflexive transitive relation.
A *reflexive ordering* (also called: “partial ordering”) is an anti-symmetric quasi-ordering.
An *equivalence* is a symmetric quasi-ordering.

We will use several binary relations $\lesssim_N$ comparing arguments according to their specificity.

**Corollary 0** *If* $\lesssim_N$ *is a quasi-ordering, then its equivalence* $\approx_N$ *is an equivalence, its ordering* $<_N$ *is an ordering, and its reflexive ordering* $\leq_N$ *is a reflexive ordering.*

# Abstract Specificity Orderings

For any relation written as $\lesssim_N$
("being more or equivalently specific w.r.t. $N$"),
we define:

$$\gtrsim_N \quad := \quad \{\ (X,Y) \mid Y \lesssim_N X\ \} \text{("less or equivalently specific")},$$

$$\approx_N \quad := \quad \lesssim_N \cap \gtrsim_N \qquad \text{("equivalently specific")},$$

$$<_N \quad := \quad \lesssim_N \setminus \gtrsim_N \qquad \text{("properly more specific")},$$

$$\leq_N \quad := \quad <_N \cup \{\ (X,X) \mid X \text{ is an argument }\} \qquad \text{("more specific or equal")},$$

$$\triangle_N \quad := \quad \left\{ (X,Y) \;\middle|\; \begin{array}{l} X,Y \text{ are arguments with} \\ X \not\lesssim_N Y \text{ and } X \not\gtrsim_N Y \end{array} \right\} \qquad \text{("incomparable w.r.t. specificity")}.$$

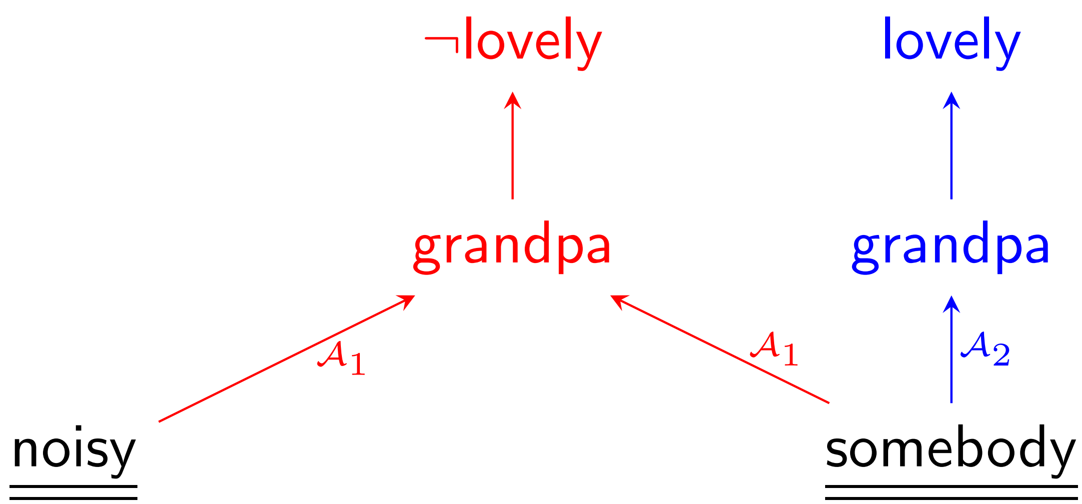
# Efficiency Considerations

- Let $H$ be a set of hypotheses. Then, roughly speaking, $(\mathcal{A}_1, L_1) \lesssim (\mathcal{A}_2, L_2)$ if every activation set $H$ for $(\mathcal{A}_1, L_1)$ is also one for $(\mathcal{A}_2, L_2)$.
- Naïve procedure enumerates possible activation sets, which is exponential in the number of possible hypothesis literals.
- Clearly, the effort for computing $\lesssim_{\mathrm{CP1}}$ is lower than that of $\lesssim_{\mathrm{P3}}$ because of $\mathfrak{T}_{\Pi} \subseteq \mathfrak{T}_{\Pi \cup \Delta}$.
- While checking $\lesssim_{\mathrm{Px}}$, attention cannot be restricted to derivations which make use only of defeasible rules given in the arguments. Therefore, [Stolzenburg&al, 2003] introduce pruning derivation trees.

# Path Characterization for Specificity

- $(\mathcal{A}_1, L_1) \leq (\mathcal{A}_2, L_2)$ if $(\mathcal{A}_1, L_1)$ and $(\mathcal{A}_2, L_2)$ are two arguments in the given specification and for each derivation tree $T_1$ for $L_1$ there is a derivation tree $T_2$ for $L_2$ such that $T_1 \trianglelefteq T_2$.
- Let $T_1$ and $T_2$ be derivation trees. Then, $T_1 \trianglelefteq T_2$ if for each $t_2 \in \mathsf{Paths}(T_2)$ there is a path $t_1 \in \mathsf{Paths}(T_1)$ such that $t_1 \subseteq t_2$ (omitting the root nodes).
- If the arguments involved in the comparison correspond to exactly one and-tree, then $\lesssim_{\mathrm{P2}}$ coincides with the path characterization ($\leq$ and $\trianglelefteq$). Cf. Example 1 of [Poole, 1985].
- Two and-trees can be compared efficiently w.r.t. $\trianglelefteq$. It requires pairwise comparison of all nodes in the trees for each path. Hence, the respective complexity is polynomial in the size of the derivation trees.

# Path vs. Argument Characterization




- Poole's specificity: $(\mathcal{A}_1, \neg\text{lovely}) <_{\text{P1--3}} (\mathcal{A}_2, \text{lovely})$
- Corrected version: $(\mathcal{A}_1, \neg\text{lovely}) <_{\text{CP1/2}} (\mathcal{A}_2, \text{lovely})$
- Path characterization: $(\mathcal{A}_1, \neg\text{lovely}) \leq (\mathcal{A}_2, \text{lovely})$
  $\big\{\{\text{noisy}, \text{grandpa}\}, \{\text{someb.}, \text{grandpa}\}\big\} \quad \big\{\{\text{someb.}, \text{grandpa}\}\big\}$
- Argument sets: $(\mathcal{A}_1, \neg\text{lovely}) \sqsubseteq (\mathcal{A}_2, \text{lovely})$
  $\big\{\{\text{noisy}, \text{somebody}\}, \{\text{grandpa}\}\big\} \quad \big\{\{\text{somebody}\}, \{\text{grandpa}\}\big\}$

# Back to the Arguments

- Characterization dual to $\trianglelefteq$ based on the rules in the arguments:
- Let $(\mathcal{A}_1, L_1)$ and $(\mathcal{A}_2, L_2)$ be two arguments and $R_i$ ($i = 1, 2$) be the set of (strict and defeasible) rule bodies used in the respective proofs. We define: $R_1 \sqsubseteq R_2$ if for all $r_1 \in R_1$ there exists an $r_2 \in R_2$ such that $r_2 \subseteq r_1$.
- Simplified version: Interpret $\sqsubseteq$ simply as subset (w.r.t. complete rule sets).
- This is close to the notion *more conservative than* [Besnard&Hunter, 2001]: $\mathcal{A}_1 \subseteq \mathcal{A}_2$ and $L_2 \vdash L_1$.
- Checking $\subseteq$ on ground literal sets can be done efficiently (NP-complete for general literals).

# Further Conclusions

- Computing $\lesssim_{\mathrm{CPx}}$ can be done by a modified SLD-resolution procedure, but has to enumerate all possible derivations for each query.
- Path ($\trianglelefteq$) and argument ($\sqsubseteq$) characterizations can be computed efficiently. However, they coincide with specificity notion only in special cases (e.g. no strict rules).
- Further investigation is required . . .
- Thank you very much for your attention!