

Gisbert Hasenjaeger and a Most Interesting Unpublished Draft for Hilbert and Bernays' "Grundlagen der Mathematik"

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Abstract¹

In 1934, in Bernays' preface to the *first edition* of the first volume of Hilbert and Bernays' monograph "Grundlagen der Mathematik", a nearly completed draft of the the finally two-volume monograph is mentioned, which had to be revoked because of the completely changed situation in the area of proof theory after Herbrand and Gödel's revolutionary results. Nothing at all seems to be known about this draft and its whereabouts.

A third of a century later, Bernays' preface to the *second edition* (1968) of the first volume of Hilbert and Bernays' "Grundlagen der Mathematik" mentions joint work of Hasenjaeger and Bernays on the second edition. Bernays states there that "it became obvious that the integration of the many new results in the area of proof theory would have required a complete reorganization of the book", i.e. that the inclusion of the intermediately found new results in the area of proof theory turned out to be unobtainable by a revision, but would have required *a complete reorganization of the entire textbook*. We will document that — even after the need for a complete reorganization had become obvious — this joint work went on *to a considerable extent*. Moreover, we will document *when Hasenjaeger stayed in Zürich* to assist Bernays in the completion of the second edition.

In May 2017, we identified an incorrectly filed text in Bernays' scientific legacy at the archive of the ETH Zurich as a candidate for the beginning of the revoked draft for the first edition or of a revoked draft for the second edition.

¹This article is a thoroughly revised and improved version of (Wirth, 2021).

In a *partial presentation* and careful investigation of this text we gather only some minor evidence that this text is the beginning of the nearly completed draft of the first edition, but ample evidence that this text is part of the work of Hasenjaeger and Bernays on the second edition. We provide some evidence that this work has covered *a complete reorganization of the entire first volume, including a completely new version of its last chapter on the ι* .

The great mathematician, logician, and computer scientist Gisbert Hasenjaeger has been completely neglected up to now by historiography — except for some autobiographical remarks and except for his work centered around computing machinery,² in particular around the German Enigma and his fight against Alan Turing. He lost that fight luckily, as a victory may have prolonged the Second World War in Germany up to the time when the US were willing to throw atom bombs. As this is to the best of our knowledge the only historiographical publication on Gisbert Hasenjaeger’s seminal work in logic and mathematics, we felt the need to include here his own Curriculum Vitae with an English translation and a few remarks on it into our Appendix and, moreover, to include his hopefully complete bibliography into our Bibliography.

1 Introduction

1.1 Hilbert’s program in general

By the 1930s, ground-breaking work had been achieved by German scientists, especially in philosophy, psychology, physics, chemistry, and mathematics. With the Nazis’ seizure of power in 1933, the historical tradition of German research was discontinued in most areas, and, as a further consequence, many achievements of German science in the first half of the 20th century have still not been sufficiently recognized. This is the case especially for those developments that had not been completed before the Nazis covered Germany under twelve years of intellectual darkness.

David Hilbert (1862–1943) is one of the most outstanding representatives of mathematics, mathematical physics, and logic-oriented foundational sciences in general (Reid, 1970). From the end of the 19th century to the erosion of the University of Göttingen by the Nazis, HILBERT formed and reshaped many areas of applied and pure mathematics. Most well-known and highly acknowledged are his “Foundations of Geometry” (Hilbert, 1899).³

After initial work at the very beginning of the 20th century, Hilbert re-intensified his research into the logical foundations of mathematics in 1917, together with his new assistant Paul Bernays (1888–1977). Supported by their PhD student Wilhelm Ackermann (1896–1962), Bernays and Hilbert developed the field of

²Cf. (Börger & Glaschick, 2022).

³Cf. (Toepell, 1986).

proof theory (or *metamathematics*), where formalized mathematical proofs become themselves the objects of mathematical operations and investigations — just as numbers are the objects of number theory. The goal of Hilbert's endeavors in this field was to prove the consistency of the customary methods in mathematics once and for all, without the loss of essential theorems as in the competing intuitionist movements of Kronecker, Brouwer, Weyl, and Heyting. The proof of the consistency of mathematics was to be achieved by sub-division into the following three tasks:

- *Arithmetization* of mathematics.
- *Logical formalization* of arithmetic.
- *Consistency proof* in the form of a *proof of impossibility*: It cannot occur in arithmetic that there are formal derivations of a formula A and also of its negation \bar{A} .

The problematic step in this program (nowadays called *Hilbert's program*) is the consistency proof.

Hilbert's program was nourished by the hope that mathematics — as the foundation of natural sciences, and in particular of modern physics — could thus provide the proof of its own groundedness. This was a paramount task of the time, not least because of the foundational crisis in mathematics (which had been evoked among others by Russell's Paradox at the beginning of the 20th century) and the vivid philosophic discussions of the formal sciences stimulated *inter alia* by Wittgenstein and the Vienna Circle.⁴

It should be recognized that Hilbert's primary goal was neither a reduction of mathematical reasoning and writing to formal logic as in the seminal work of Whitehead & Russell (1910–1913), nor a formalization of larger parts of mathematics as in the publications of the famous French Bourbaki (1939ff.) group of mathematicians. His ambition was to secure — once and for all — the foundation of mathematics with consistency proofs, in which an intuitively consistent, "finitist" part of mathematics was to be used for showing that no contradiction could be formally derived in larger and larger parts of non-constructive and axiomatic mathematics.

Hilbert's program fascinated an elite of young outstanding mathematicians, among them John von Neumann (1903–1957), Kurt Gödel (1906–1978), Jacques Herbrand (1908–1931), and Gerhard Gentzen (1909–1945), whose contributions essentially shaped the fields of modern mathematical logic and proof theory.

We know today that Hilbert's quest to establish a foundation for the whole scientific edifice could not be successful to the proposed extent: Gödel's incompleteness theorems dashed the broader hopes of Hilbert's program. Without the emphasis that Hilbert has put on the foundational issues, however, our negative and positive knowledge on the possibility of a logical grounding of mathematics (and thus of all exact sciences) would hardly have been achieved at his time.

⁴Cf. e.g. (Wittgenstein, 1994), (Waismann, 1967).

1.2 Drafts for Hilbert–Bernays

The central and most involved presentation of Hilbert’s program and Hilbert’s proof theory is found in the two-volume monograph “Grundlagen der Mathematik” of Hilbert & Bernays (1934; 1939), and its second revised edition (Hilbert & Bernays, 1968; 1970).

We should not forget the historical context of the original writing of these texts in the late 1920s and the 1930s: First, Herbrand’s Fundamental Theorem and Gödel’s incompleteness theorems hit the new field of proof theory like a hurricane. Moreover, after the Nazi takeover of Germany in January 1933, Bernays was expelled from his academic position in Göttingen in April 1933, and had to leave the country. As a consequence, both volumes show strong signs of reorganization and rewriting, sometimes even the signs of hurry to meet the publication deadlines.

Because of this editorial and historical context, the drafts for both editions of the two Hilbert–Bernays volumes are of special interest, in particular the drafts that did not find their way into any of the editions.

Until recently, however, we did not know anything substantial about these revoked drafts for Hilbert–Bernays. In May 2017, however, we identified an incorrectly filed text in Bernays’ scientific legacy in the archive of the ETH Zurich as a candidate for such a revoked draft. We will describe and investigate this text in this paper, and discuss at what time it was probably written, and by whom, &c.

There are two infamous revoked drafts for Hilbert–Bernays of which Bernays stated that they at least had existed — one for the first and one for the second edition:

1.2.1 The Mentioning of the Draft for the First Edition

Bernays’ “Vorwort zur ersten Auflage”⁵ of (Hilbert & Bernays, 1934, p.VII f.), begins as follows:

“Eine Darstellung der Beweistheorie, welche aus dem HILBERTSchen Ansatz zur Behandlung der mathematisch-logischen Grundlagenprobleme erwachsen ist, wurde schon seit längerem von HILBERT angekündigt.

Die Ausführung dieses Vorhabens hat eine wesentliche Verzögerung dadurch erfahren, daß in einem Stadium, in dem die Darstellung schon ihrem Abschluß nahe war, durch das Erscheinen der Arbeiten von HERBRAND und GÖDEL eine veränderte Situation im Gebiet der Beweistheorie entstand, welche die Berücksichtigung neuer Einsichten |VIII zur Aufgabe machte. Dabei ist der Umfang des Buches angewachsen, so daß eine Teilung in zwei Bände angezeigt erschien.”

⁵“Preface to the First Edition”

In the translation of (Hilbert & Bernays, 2017a, p.VII.b, engl.) (comments omitted):

“Some time ago, HILBERT announced a presentation of the proof theory that developed from the HILBERTian approach to the problems in the foundations of mathematics and logic.

The execution of this enterprise received considerable delay because the whole field of proof theory was changed by the publication of the works of HERBRAND and GÖDEL when our work was already close to completion; and this change put the consideration of new insights |VIII onto the agenda. As a consequence of this, the size of the book grew to the extent that a separation into two volumes seemed appropriate.”

Nothing about this work “already close to completion” and its whereabouts seems to be known.

1.2.2 The Mentioning of the Given-Up Work on the Second Edition

Bernays' “Vorwort zur zweiten Auflage”⁶ of (Hilbert & Bernays, 1968, p.V) begins as follows:

“Schon vor etlichen Jahren haben der verstorbene HEINRICH SCHOLZ und Herr F. K. SCHMIDT mir vorgeschlagen, eine zweite Auflage der ‚Grundlagen der Mathematik‘ vorzunehmen, und Herr G. HASENJAEGER war auch zu meiner Unterstützung bei dieser Arbeit auf einige Zeit nach Zürich gekommen. Es zeigte sich jedoch bereits damals, daß eine Einarbeitung der vielen im Gebiet der Beweistheorie hinzugekommenen Ergebnisse eine völlige Umgestaltung des Buches erfordert hätte. Erst recht kann bei der jetzt vorliegenden zweiten Auflage, zu der wiederum Herr F. K. SCHMIDT den Anstoß gab, nicht davon die Rede sein, den Inhalt dessen, was seither in der Beweistheorie erreicht worden ist, zur Darstellung zu bringen.”

In the translation of (Hilbert & Bernays, 2017a, p.V, engl.):

“A number of years ago, the late HEINRICH SCHOLZ and Mr. F. K. SCHMIDT suggested the undertaking of a second edition of the ‘Foundations of Mathematics’; and moreover, to assist me in this work, Mr. G. HASENJAEGER came to Zürich for some time. Already back then, it became obvious that the integration of the many new results in the area of proof theory would have required a complete reorganization of the book. Furthermore, the present second edition (the impetus for which came again from Mr. F. K. SCHMIDT) can by no means present the substance of the achievements in proof theory since the appearance of the first edition.”

⁶“Preface to the Second Edition”

In §4.2, we will document when Gisbert Hasenjaeger came to Zürich for this purpose, and provide some evidence that the work of Hasenjaeger and Bernays has covered *a complete reorganization of the entire first volume*, which means that the main document of this collaboration and its whereabouts still remain completely unknown, and also that Bernays may give us a wrong impression by using the subjunctive “erfordert hätte”⁷ in the above quotation.

2 Our Typescript

The unpublished typescript we discuss in this paper will briefly be called “our typescript”.

2.1 Form, Location, and Incorrect Filing of Our Typescript

Its *outer form* is as follows: It is an untitled typescript, with corrections by Bernays’ hand. Our typescript has 34 pages, with page numbers 2–34 on the respective page headers. Its spelling is German–Austrian, and it includes the German letter “ß” not found on typewriters with Swiss layout. In this context it may be relevant that — according Ludwig Bernays — his close uncle and legator Paul Bernays never had a typewriter and is not known to have ever used one.⁸ The *location* of our typescript is the archive (ETH-Bibliothek, Hochschularchiv) of the ETH Zurich (Swiss Federal Institute of Technology in Zürich) (Switzerland).

The folder in which our typescript was found there by Claus-Peter Wirth on May 12, 2017, was one of two folders in the legacy of Paul Bernays under the label “Hs 973:41”, which is listed in the inventory (Bernays, 1986) on page 7 as

“41. Texte und Korrekturen zur Neuauflage des “Grundlagenbuches”
Bd. II von David Hilbert und Paul Bernays. 1970 2 Mappen”

“41. Texts and corrections for the new edition of the “Foundations
Book” Vol. II by David Hilbert und Paul Bernays. 1970 2 folders”

This was a wrong place for our typescript because its contents refer exclusively to Vol. I, published in 1934 (1st edn.) and in 1968 (2nd edn.) — neither to Vol. II, nor to the year 1970. Thus, regarding its subject, our typescript was *incorrectly filed*; since June 22, 2021, however, it is correctly filed in the new sub-folder Hs 973:41.1.

⁷“would have required”

⁸“Paul Bernays hat keine Schreibmaschine benutzt”, private E-mail to Claus-Peter Wirth from Ludwig Bernays, Zürich, the nephew and heir to the copyrights of Paul Bernays, Nov. 12, 2017.

2.2 Two Additional Copies of Our Typescript, With Footnotes!

Moreover, in January 2018, in the legacy of Gisbert Hasenjaeger,⁹ his daughter Beate Becker succeeded in finding two carbon copies of our typescript.

Both of these carbon copies come without the corrections *by Bernays' hand* found in our typescript, but one of them has most of these corrections added (mostly with a typewriter), and the other includes the typewriting of even more of the corrections by Bernays' hand found in our typescript.

One of these carbon copies comes with two extra typewritten pages containing all footnotes for our typescript, again corrected by Bernays' hand. Our typescript, however, does not contain any footnotes at all, but only raised closing parentheses (without any numbers, letters, asterisks or other signs) to indicate their respective positions in the text.

Regarding the font and the actual typing of the letters, we did not find any significant differences between our typescript and the extra footnote pages found by Beate Becker.

2.3 Presentational Form of Our Excerpts

The deletions and additions by Bernays' handwritten remarks are not documented in the following excerpts from our typescript; only the version of our typescript after the application of Bernays' hand-written corrections is presented here.

Therefore, the additions ([this text is added]) indicated in these excerpts are ours, not Bernays'. As there are only two occurrences of required deletion in these excerpts, we have just indicated this by means of footnotes.¹⁰

Moreover — in these excerpts — the footnotes of the form [...] are our additions, whereas the other footnotes are those from the two extra pages found with one of the two carbon copies of our typescript in the Hasenjaeger legacy (cf. § 2.2).

All footnote numbers in these excerpts are introduced by us. This form of presentation seems appropriate in particular because our typescript has no footnotes (but only raised closing parentheses to indicate their positions, cf. § 2.2).

The positions of the original page breaks in our typescript are indicated by the sign “|” in our excerpts, with the number of the new page in a lower index, such as “|₅” for the page break from page 4 to page 5.

If we use small capitals for a name (as in “HILBERT”) this is to indicate a full capitalization in our typescript (as in “HILBERT”, even if it is written “Hilbert” because of a typo) or small capitals in (Hilbert & Bernays, 1934; 1939; 1968; 1970).

⁹By the end of the year 2018, Beate Becker gave her part of the scientific legacy of her father Gisbert Hasenjaeger in 9 boxes to the following archive: Legacy of GIBERT HASENJAEGER, Deutsches Museum, München, Archiv: “Nachlässe H, NL 288”.

¹⁰Cf. Notes 19 and 31.

2.4 Discussion and Excerpts of § 1 of Our Typescript

The first section of our typescript has the headline “Einleitung.”¹¹ and the additional subsection headline “§ 1. Einführung in die Fragestellung”.¹²

2.4.1 Parallel introductory part, several paragraphs missing in our typescript

The text starts literally with the first paragraph of § 1 of (Hilbert & Bernays, 1934; 1968) (p. 1), including the enumerated list of three items.

The next paragraph of (Hilbert & Bernays, 1934; 1968), however, is not present in our typescript, namely the paragraph on the “verschärften methodischen Anforderungen”¹³ resulting in “eine neue Art der Auseinandersetzung mit dem Problem des Unendlichen”.¹⁴

Then, however, the text continues almost literally with the penultimate paragraph and the first sentence of the last paragraph of p. 1 of (Hilbert & Bernays, 1934; 1968).

The part on the subject of the “*existential Form*”¹⁵ (which starts in the middle of the last paragraph of p. 1 and runs up to the end of the 1st paragraph starting on p. 2) is again missing in our typescript – just as the introduction to this subject was omitted before, namely in the paragraph on the problem of the infinite, right after the enumerated list of three items. Instead of this part, the paragraph ends in our typescript with a digression into satisfiability and consistency.

Then the last paragraph on p. 2 of our typescript follows almost literally the text from the 2nd paragraph starting on p. 2 of (Hilbert & Bernays, 1934; 1968) up to the end of the 2nd paragraph starting on p. 3 of (Hilbert & Bernays, 1934; 1968), where only the last part of the last sentence

“als gültig vorausgesetzt, und wir kommen so zu der Frage, welcher Art diese Geltung ist.”¹⁶

is missing in our typescript, where the sentence ends with

“zugrundegelegt.”¹⁷

instead — at the *very end of page 4* of our typescript.

¹¹“Introduction.”

¹²“§ 1. Introduction to the Problem Definition”

¹³“sharpened methodological requirements”

¹⁴“to deal with the problem of the infinite in a new way”

¹⁵“*existential form*”

¹⁶“presupposed to be valid. And so we come to the question what nature of validity this is.”

¹⁷“taken as a basis.”

2.4.2 Over 16 pages entirely missing in the typescript

The remainder of §1 of (Hilbert & Bernays, 1934; 1968) is entirely omitted in our typescript, i.e. everything from the penultimate paragraph on p. 3 to the very end of p. 19 is entirely missing. This remainder covers – among others subjects – the following: the logical symbolism, axiomatizations of geometry, satisfiability and universal validity of formulas, the ACHILLES Paradox, the existence of an infinite manifold, the method of arithmetization, and the task of a proof of consistency as an impossibility proof.

2.4.3 A completely unknown introduction; from our typescript, pp. 5–14

In §1 of our typescript, however, right after the very end of page 4 (just mentioned at the very end of §2.4.1), there is a completely unknown, most interesting, and well-written introduction to the foundations of mathematics, HILBERT's proof theory, and the finitist standpoint, which seems to be entirely unpublished up to now.

We now present this introduction in the following way: The German original is found on the pages with even page numbers and our English translation on the respectively following pages with odd page numbers.

To this end, the remainder of the current page is left blank here.

|₅ Wir kommen somit zu dem zweiten der anfangs genannten Themata der Grundlagenuntersuchungen. In der Begründung der Analysis ist es ja im 19. Jahrhundert zuerst durch die Untersuchungen von BOLZANO und CAUCHY und hernach deren Weiterführung und Vollendung durch DEDEKIND, CANTOR und WEIERSTRASZ gelungen,¹⁸ die Methoden der Infinitesimalrechnung, die ja in ihren Anfängen einer vollen Deutlichkeit entbehrten und mehr nur instinktiv gehandhabt wurden, im Sinne einer stärkeren Anknüpfung an die klassischen Methoden der griechischen Mathematiker EUDOXOS und ARCHIMEDES zu einer¹⁹ präzise mitteilbaren und lehrbaren zu gestalten.

Indem diese Deutlichkeit erreicht wurde, traten zugleich die zugrundeliegenden methodischen Voraussetzungen mehr hervor, und man ging auch dazu über, diese Voraussetzungen über die Zielsetzung der Infinitesimalrechnung hinaus systematisch zu verwerten, wie es ja vor allem in der CANTOR'schen Mengenlehre geschah. Die hier stattfindende starke Überschreitung des mathematisch Gewohnten weckte vielerseits Kritik, die dann noch durch die Entdeckung der mengentheoretischen Paradoxien bestärkt wurde.

Wenngleich es sich nun auch bei näherem Zusehen erwies, daß es zur Verhütung der Paradoxien genügte, gewisse extreme Begriffsbildungen zu vermeiden, die tatsächlich für den Aufbau der Mengenlehre und erst recht für die Methoden der Analysis gar nicht erforderlich sind, so ist doch seitdem die Diskussion über die Grundlagen der Mathematik nicht zur Ruhe gekommen, und man hat sich auch jener Paradoxien als Argument bedient, um viel weiter gehende Einschränkungen des mathematischen Verfahrens zu motivieren, als sie zur Behebung der Widersprüche |₆ direkt erfordert werden.

Für eine gründliche Stellungnahme zu dieser Grundlagendiskussion erscheint eine eingehende Betrachtung der *logischen Struktur der mathematischen Theorien* als geboten.

¹⁸MÉRAY, Enc. I.1, 3 (1904).

¹⁹[This word "einer" should have been deleted by Bernays. Indeed, "einer stärkeren Anknüpfung" and "einer vollen Deutlichkeit" are the most salient referents for "präzise mitteilbaren und lehrbaren", which do not make any sense, however. Thus, "die Methoden der Infinitesimalrechnung" is the only possible referent, which is a plural, whereas "einer präzise mitteilbaren und lehrbaren" is singular. On the basis of our wide experience with Bernays's scientific writing we are certain that Bernays would definitely have disambiguated this to "einer präzise mitteilbaren und lehrbaren Methode". It cannot be the case, however, that "Methode" is omitted in the typescript, because Bernays would never have claimed that infinitesimal calculus is to be taught only by a single method. All in all, "einer" must have remained in the typescript by error.]

|₅ Let us now turn to the second topic of the foundational investigations listed at the beginning, regarding the grounding of Analysis. As is well known, in the 19th century, first through the investigations of BOLZANO and CAUCHY and then through their continuation and completion by DEDEKIND, CANTOR and WEIERSTRASZ,²⁰ the methods of infinitesimal calculus, which lacked clarity and were applied more or less instinctively in their beginnings, were given a precisely communicable and teachable form, in the sense of a closer orientation toward the classical methods of the Greek mathematicians EUDOXOS and ARCHIMEDES.

As this clarity was achieved, the underlying presuppositions of these methods became more obvious as well. Moreover, these presuppositions were then systematically applied beyond the field of infinitesimal calculus, in particular in CANTOR's set theory. The strong transgression of the common usage of these presuppositions in mathematics aroused criticism which came from many sides and was then further encouraged by the discovery of the set-theoretic paradoxes.

On a closer inspection, however, it turned out that the paradoxes can be averted by the exclusion of certain extreme concept formations, which are not actually required for the constructions of set theory, and even less for the methods of Analysis. And yet, the discussion on the foundations of mathematics has not settled down since then. Moreover, those paradoxes were employed as motivational arguments for restrictions on the mathematical approach that go far beyond of what is immediately required for eliminating the contradictions. |₆

A careful positioning in this discussion on the foundations of mathematics requires a more detailed consideration of the *logical structure of the mathematical theories*.

²⁰MÉRAY, Enc. I.1, 3 (1904).

[It is difficult to say what this abbreviated footnote text by Bernays' hand means, but it probably refers to section "6. Point de vue de CH. MÉRAY"; pp.147–149 in (Pringsheim, 1904/07). This would make sense because our typescript omits the first name to be mentioned in the list, namely Charles Méray (1835–1911), and (Pringsheim, 1904/07) is one of the first texts that mentions the primacy of this neglected mathematician. Note that the German original (Pringsheim, 1898) does not contain this section on Méray at all.

The footnote is not executed in any of the two carbon copies (cf. § 2.2) of our typescript, but Bernays probably wanted to add "MÉRAY," before "DEDEKIND" and have the expanded proper citation in a footnote.]

In der Tat bemerkt man, daß es sich bei den zur Diskussion stehenden Verfahren der Mathematik um Methoden des Folgerns und der Begriffsbildung handelt, daß also hier eine Art der Erweiterung der gewöhnlichen Logik zur Geltung kommt. Zugleich zeigt sich eine enge Verflochtenheit des Mathematischen mit dem Logischen: einerseits tritt die Mengenlehre ihrem Gegenstand nach, durch die Beziehung von Mengen und Prädikaten (d. h. durch das Verhältnis von Umfang und Inhalt der Begriffe) in engste Berührung mit der Logik; andererseits wird man in der systematischen Untersuchung der logischen Bildungsformen und Schlußweisen mit Notwendigkeit auf mathematische Betrachtungen geführt. So ist ja bereits die traditionelle Lehre von den kategorischen Schlüssen eine typisch mathematische Untersuchung, was nur durch ihre historische Einordnung in die Philosophie leicht verdeckt wird. Mit dieser mathematischen Seite des Logischen hängt es auch zusammen, daß die logischen Schlüsse – wie sie insbesondere bei der reichhaltigeren Anwendung der Logik in den mathematischen Theorien zur Verwendung kommen –, in einer mathematischen Weise fixierbar und aus einer Reihe von wenigen Elementarprozessen zusammensetzbar sind.

Dieser Sachverhalt wurde zur vollen Deutlichkeit gebracht |₇ durch die Entwicklung der Systeme der symbolischen Logik, wie sie, vorbereitet durch den BOOLE'schen Logikkalkül,²¹ um die Jahrhundertwende insbesondere von PEIRCE, FREGE, SCHRÖDER, PEANO, WHITEHEAD u. RUSSELL geschaffen wurden. Bei der Konstruktion dieser Systeme ging man teils darauf aus, eine handliche Symbolik zu gewinnen, die zugleich eine genauere Kontrolle der Schlußfolgerungen ermöglichte, teils bezweckte man eine Einordnung der Mathematik in die Logik.

Es war der Gedanke HILBERTS, die logische Symbolik dazu zu verwenden, die mathematischen Beweismethoden zum Gegenstand einer mathematischen Untersuchung, einer „Beweistheorie“, zu machen.²² Der wesentliche Gesichtspunkt dabei ist, die Methode der formalen Axiomatik auch auf das logische Schließen selbst, wie es in den Theorien der Arithmetik und Mengenlehre ausgeübt wird, anzuwenden und somit an die Stelle der Prozesse der logische[n] Begriffsbildung und Folgerung formal angesetzte Operationen treten zu lassen. Hierdurch gewinnen wir den Vorteil, daß wir bei strittigen Begriffen und Schlußweisen nicht die inhaltliche Bedeutung in Betracht zu ziehen brauchen, sondern nur den formalen Effekt, der durch ihre Anwendung in den deduktiven Prozessen bewirkt wird. Dieser Effekt läßt sich vom Standpunkt einer ganz elementaren Betrachtung verfolgen. Wir haben so die Möglichkeit, Methoden, die vom inhaltlichen Standpunkt problematisch erscheinen, als beweistechnische Verfahren zu akzeptieren und zu rechtfertigen.

²¹[*Sic!*]

²²Der erste Ansatz in dieser Richtung war der HILBERT'sche Heidelberger Vortrag 1904 (Hilbert, 1905), der freilich noch ganz im Fragmentarischen blieb. (In diesem wurde auch schon der Gedanke eines gemeinsamen Aufbaus von Mathematik und Logik zur Geltung gebracht.) Eine erste Weiterführung dieser Gedanken findet sich, noch vor HILBERTS späteren Untersuchungen, in dem Werk von JULIUS KÖNIG: Neue Grundlagen der Logik, Arithmetik, und Mengenlehre (Leipzig, 1914) [(König, 1914)].

Let us notice that the mathematical approaches under discussion, in fact, consist of methods of inference and concept formation. This means that some extension of the usual logic comes to bear in these approaches. At the same time, we notice that the mathematical and the logical are tightly intertwined here: On the one hand, considering its objects, set theory comes very close to logic through the relation between sets and predicates, i.e. through the relationship between extension and content of notions. On the other hand, any systematic investigation of the modes of logical formation and inference will necessarily lead us to mathematical considerations. As a matter of fact, the traditional teaching of the categorical inferences is already a typical mathematical investigation; this fact is only superficially concealed by the historical classification of this teaching into philosophy. It is also connected with this mathematical side of the logical that the logical inferences — in particular as they occur in comprehensive logic application in mathematical theories — can be captured in a mathematical way as being composed of a small number of elementary processes.

This state of affairs was brought to full clarity |₇ by the development of the systems of symbolic logic, which, prepared by the BOOLEAN logic calculus, were created around the turn of the century in particular by PEIRCE, FREGE, SCHRÖDER, PEANO, WHITEHEAD & RUSSELL. The construction of these systems partly aimed at obtaining a handy symbolism, which also facilitated a more precise control over the conclusions, and partly aimed at a classification of mathematics into logic.

It was HILBERT's idea to create a "proof theory", in which the mathematical proof methods, formalized in a logical symbolism, become the objects of mathematical investigation.²³ The significant viewpoint is to apply the method of formal axiomatics also to the logical reasoning itself as it is applied in the theories of arithmetic and set theory, and thus to let formally specified operations take the places of the processes of logical concept formation and conclusion. This has the advantage that we need not consider the contentual meaning of contentious notions and modes of inference, but only the formal effect of their application in deductive processes. This effect can be traced by means of most elementary considerations. We thus have the possibility of accepting and justifying methods as proof-technical procedures, no matter whether they are problematic from the contentual standpoint.

²³The first approach in this direction was HILBERT's Heidelberg talk of 1904 (Hilbert, 1905), which, of course, had to remain outright fragmentary. (In this talk also the idea of a joint construction of mathematics and logic is already brought to bear.) A first continuation of these ideas can be found, even before HILBERT's later investigations, in the work of JULIUS KÖNIG: *Neue Grundlagen der Logik, Arithmetik, und Mengenlehre* [New Foundations of Logic, Arithmetic, and Set Theory] (Leipzig, 1914) [(König, 1914)].

In diesem Sinne hat HILBERT die Aufgabe gestellt und in Angriff genommen, das System der Analysis und Mengenlehre als $\S 8$ ein widerspruchsfreies Gedankengebäude zu erweisen. Diese Aufgabe gliedert sich in zwei Teile.

Es handelt sich zunächst darum, die Beweismethoden der Analysis und Mengenlehre einer formalen Axiomatik zu unterwerfen oder, wie wir es kurz nennen wollen, zu *formalisieren*. Hierfür konnte sich HILBERT auf die bereits ausgebildeten zuvor genannten Systeme der Logistik stützen, in denen eine solche Formalisierung bereits geleistet war. Der Gesichtspunkt der streng formalen Deduktion wurde zuerst bei FREGE scharf herausgestellt und für Teile der Mathematik zur Durchführung gebracht. Die Methode zur handlichen Ausgestaltung einer Formalisierung wurde durch PEANO entwickelt. Eine Verbindung von beiden fand in den *Principia Mathematica* durch WHITEHEAD und RUSSELL statt.

Die hier[mit] vorliegende Formalisierung ist freilich für die Zwecke der Beweistheorie insofern nicht vorteilhaft, als sie keine Gliederung in elementarere und höhere Bereiche der Begriffsbildung und des Schließens ermöglicht. Das rührt davon her, daß in den *Principia Mathematica* sowie bei FREGE die Gewinnung der Zahlentheorie aus der allgemeinen Mengenlehre als eines der Hauptziele genommen ist. So können hier die Methoden einer elementareren Behandlung der Zahlentheorie nicht in Erscheinung treten.

Von einer Formalisierung für die Zwecke der Beweistheorie ist zu wünschen, daß sie eine analoge axiomatische Gliederung der logisch-mathematischen Bildungen und Prozesse liefert, wie sie in der üblichen Axiomatik durch die Sonderung der Axiomengruppen bewirkt wird. Unter diesem Gesichtspunkt erscheint ein schichtweiser Aufbau des deduktiven Formalismus als angemessen. |9

So wurde man veranlaßt, von neuem die Formalisierung der mathematischen Disziplinen vorzunehmen, und man ist dabei zur präzisen Beschreibung von naturgemäß abgegrenzten Teilbereichen der logisch-mathematischen Deduktion gelangt, welche sich als solche zum ersten Mal an Hand des Formalisierungsprozesses darstellten.

Auf Grund der vollzogenen Formalisierung gewinnt nun die Aufgabe des Nachweises der Widerspruchsfreiheit für die arithmetischen Disziplinen eine bestimmtere mathematische Form. Es handelt sich jetzt darum, zu erkennen, daß die festgelegten Prozesse der Aussagenbildung und der Schlußfolgerung nicht zur Herleitung solcher Sätze führen können, die einander im Sinne der gewöhnlichen inhaltlichen Interpretation widersprechen. Das Widersprechen von Sätzen stellt sich mittels der Formalisierung der Negation durch eine einfache Beziehung der entsprechenden Satzformeln dar.

In this sense, HILBERT has set and tackled the task of proving the system of Analysis and set theory to be \models_8 a consistent construct of ideas. This task subdivides into two parts.

First we have to represent the proof methods of Analysis and set theory in a formal axiomatics or, as we want to call it briefly, to *formalize* them. To this end, HILBERT could resort to the mentioned full-fledged systems of formal mathematical logic, in which such a formalization had already been accomplished. FREGE was the first who clearly displayed the viewpoint of strict formal deduction and put it into practice for parts of mathematics. It was PEANO who first developed handy presentations for such formalizations. Both accomplishments were combined in the *Principia Mathematica* by WHITEHEAD and RUSSELL.

This formalization is not advantageous for the purposes of proof theory, however, as it lacks subdividability into elementary and more advanced domains of concept formation and inference. This lack of subdividability has its historic origin in one of the main goals of the Principia Mathematica as well as FREGE's work: the extraction of number theory from general set theory. Consequently, methods for a more elementary treatment of number theory cannot appear in these formalizations.

It is to be desired from a formalization for the purposes of proof theory that it admits an axiomatic subdivision of the logico-mathematical formations and processes that is analogous to the effect of the separation of axioms into different groups, as it is often found in customary axiomatics. From this viewpoint, it appears to be adequate to construct the deductive formalism in layers. \models_9

Therefore, the formalization of the mathematical disciplines was put on the agenda again. And, in the course of the resulting formalization process, naturally separated subdomains of logico-mathematical deduction were brought into separate being for the first time, together with their precise descriptions.

On the basis of the given formalization, the task of proving the consistency of the arithmetical disciplines now gains a more definite mathematical form. It now means to realize that the uniquely defined processes of conclusion and of proposition formation cannot lead to the derivation of sentences that contradict each other according to the common contentual interpretation. By means of the formalization of negation, a contradiction of sentences can be represented simply as the occurrence of two corresponding formalized sentences.²⁴

²⁴[This sentence may mean two different things; in the order of preference: 1. "The realization of sentences that contradict each other can be reduced to tracing the derivation of two sentences of the forms A and \bar{A} ." Soundness of this problem reduction requires that the derivation process satisfies *ex falso quodlibet*, and completeness of the reduction requires a certain form of completeness of the derivation process. The latter, for instance, is not given for the derivation process in (Wirth & Stolzenburg, 2016, §2.2). 2. "Contradiction may be defined in this context as two derivable sentences of the forms A and \bar{A} ."]

Überdies ergibt sich bei einer zweckmäßigen Formalisierung der Aussagenlogik noch eine Vereinfachung in der Weise, daß ausgehend von irgend einem Widerspruch jede beliebige Aussage des formalisierten Bereichs herleitbar wird. Auf Grund davon genügt es, um die Unmöglichkeit eines Widerspruchs in dem genannten Sinne erkennen zu lassen, daß man das Gegenteil eines bestimmten einzelnen elementar gültigen Satzes als nicht herleitbar erweist. Die Ausführung eines solchen Nachweises – anschließend an den schrittweisen Aufstieg der Teilbereiche – bildet den zweiten Teil des HILBERT'schen Programms. Die Durchführung ist freilich HILBERT nicht gelungen, und es ist auch heute noch nicht abzusehen, ob – oder vielmehr in welchem Sinne – sie gelingen kann. |₁₀

Es bestehen nämlich hier nicht nur große technische, sondern auch grundsätzliche Schwierigkeiten. Diese erheben sich vor allem mit Bezug auf die Frage, welche Mittel für den gewünschten Nachweis zugelassen werden sollen. In der Tat ging ja das Bedürfnis für einen solchen Nachweis von einer Kritik der üblichen Beweismethoden aus. Soll dieser Kritik Rechnung getragen werden, so darf der Nachweis der Widerspruchsfreiheit nicht seinerseits auf einer Verwendung der kritisierten Methoden beruhen.

Durch diese Erwägung erhalten wir aber zunächst nur eine Abgrenzung im negativen Sinne, und es bleibt noch die Aufgabe, genauer zu bestimmen, auf welche Arten der Überlegung die Beweistheorie sich stützen soll. Für die Wahl dieses Standpunkts wird uns ein Anhalt durch das Erfordernis gegeben, daß zumindest ja die Behauptung der Widerspruchsfreiheit für die formalisierten Theorien sich präzise fassen lassen muß.

Diesem Erfordernis wird bereits genügt durch eine Art der elementaren mathematischen Betrachtungsweise, welche HILBERT als den *finiten Standpunkt* bezeichnet hat. Es ist diejenige Art anschaulicher mathematischer Überlegung, wie sie in der elementaren Kombinatorik angewandt wird. Auch die elementare Zahlentheorie und Buchstaben-Algebra läßt sich auf diese Art behandeln. Das Kennzeichnende für die finite Betrachtung besteht in folgenden Momenten:

Moreover, there is a further simplification by the fact that, for a formalization of propositional logic appropriate for our purposes, an arbitrary contradiction will admit the derivation of each and every proposition of the formalized domain. To realize the impossibility of a contradiction in the mentioned sense, it thus suffices to pick any definite, elementary valid sentence and show that the opposite of this sentence cannot be derived. After the stepwise construction of the subdomains, it is the execution of such a proof which forms the second part of HILBERT's program. Admittedly, this execution was not achieved by HILBERT, and even today it is still not foreseeable whether — or rather in which sense — it may be achieved. |₁₀

In fact, there are not only major technical difficulties here, but also fundamental ones. These arise mainly with regard to the question, which means are to be admitted for the desired proof. It was, after all, a critique of the customary proof methods that prompted the demand for such a proof. If this critique is to be taken into proper account, then the proof of consistency must not in turn rely on the application of the criticized methods.

By this consideration, however, we get only a first boundary in the negative sense; and the task still remains to determine more precisely, on which sorts of consideration proof theory is to be based. A hint on the choice of this standpoint is given by the requirement that at least the assertion of consistency must be precisely expressible for the formalized theories.

This requirement is already satisfied by some elementary mode of mathematical consideration, which HILBERT called the *finitist standpoint*. It is that mode of intuitive mathematical consideration which is applied in elementary combinatorics. Also elementary number theory and algebra can be treated in this mode. The characteristics of finitist consideration are given by the following moments:

1. Als Gegenstände werden nur *endliche* Gebilde genommen, an denen auch nur *diskrete* Gestaltsmerkmale unterschieden werden.
2. Die Formen des allgemeinen und des existentialen Urteils kommen nur auf eine eingeschränkte Art zur Anwendung, im Sinne |₁₁ der Vermeidung der Vorstellung von unendlichen Gesamtheiten; nämlich das allgemeine Urteil wird nur in hypothetischem Sinne gebraucht, als eine Aussage über jedweden vorliegenden Einzelfall, und das existentielle Urteil als ein (zweckmäßig zu vermerkender) Teil einer näher bestimmten Feststellung, in der entweder ein bestimmt strukturiertes Gebilde vorgewiesen oder ein allgemeines Verfahren aufgezeigt wird, nach dem man zu einem (gewisse Bedingungen erfüllenden) Gegenstand²⁵ einen anderen Gegenstand mit verlangten Eigenschaften gewinnen kann.
3. Alle Annahmen, die man einführt, beziehen sich auf endliche Konfigurationen.

Mit der genaueren Beschreibung und Erörterung der finiten Betrachtungsweise werden wir uns noch des Näheren zu befassen haben.

Es wäre zweifellos sehr befriedigend, wenn wir uns in der beweistheoretischen Untersuchung völlig an diesen Rahmen elementarer Betrachtung halten könnten. Die Möglichkeit hierfür scheint zunächst insofern gegeben zu sein, als ja mit Bezug auf eine formalisierte Theorie die Behauptung ihrer Widerspruchsfreiheit sich nach dem vorhin Bemerkten in finiter Form dahin aussprechen läßt, daß ein jeder formalisierte Beweis eine Endformel hat, die verschieden ist von der Negation einer bestimmten[,] geeignet gewählten Satzformel.

Die Formulierbarkeit eines Problems im Rahmen gewisser Ausdrucksmittel bietet aber noch keine Gewähr dafür, daß seine Lösung sich mit diesen Mitteln bewerkstelligen läßt. Tatsächlich hat es sich im Fall der Beweistheorie herausgestellt, daß für die gewünschten Nachweise der Widerspruchsfreiheit formalisierter Theorien die finiten Methoden nicht zulänglich sind. |₁₂ Man hat sich so genötigt gesehen, für die Beweistheorie den ursprünglichen finiten Standpunkt zu einem „konstruktiven“ Standpunkt zu erweitern, der sich etwa so kennzeichnen läßt, daß von den drei soeben genannten Forderungen nur die ersten beiden aufrechterhalten werden, die dritte aber fallen gelassen wird. So kommt man dazu, für die Beweistheorie eine ungefähr solche methodische Haltung einzunehmen, wie sie der BROUWER'sche Intuitionismus für die Mathematik überhaupt als einzig zulässig ansieht.²⁶

²⁵[This must be the plural in general: “zu (gewisse Bedingungen erfüllenden) Gegenständen”.]

²⁶Eine präzise Vergleichung ist darum hier nicht zu verlangen, weil die intuitionistische Haltung nicht durch Gebrauchsregeln, sondern durch eine philosophische Einstellung charakterisiert ist. Das wird auch von HEYTING ungeachtet der von ihm durchgeführten Formalisierung der intuitionistischen Logik und Mathematik hervorgehoben. Für eine Konfrontierung der Methoden der konstruktiven Beweistheorie mit denen des Intuitionismus sind auch die neueren Untersuchungen von G. F. C. GRISS über die negationsfreie intuitionistische Mathematik von Belang, bei welchen das Operieren mit irrealen (unerfüllten) Annahmen grundsätzlich vermieden wird (Proc. Kon. Ned. Adad. v. Wetensch. 49 (1946) [(Griss, 1946)], 53 (1950) [(Griss, 1950)] und 54 (1951) [(Griss, 1951a; 1951b; 1951c; 1951d)]).

1. The objects must be *finite* entities, distinguished only by their *discrete* attributes.
2. Universal and existential judgments may occur only if \mid_{11} they do not refer to the conception of any infinite totality: The universal judgment may be used only in the hypothetical sense, as a proposition on any individual object to be given. Moreover, the existential judgment may occur only as (an appropriately noted) part of a more specifically determined statement, which additionally presents either an entity of definite structure, or else a general procedure for generating an object of the asserted properties from any object[s] to be given (satisfying certain conditions).²⁷
3. Assumptions may be introduced only if they refer to finite configurations.

We will have to take a closer look on a more detailed description and discussion of the finitist mode of consideration later.

There can be no doubt that it would be very fulfilling if we were able to confine our proof-theoretic investigations to this framework of elementary consideration. At first, this seems to be possible insofar as — according to what we have previously noticed — the assertion of the consistency of a formalized theory can be expressed in finitist form by the assertion that every formalized proof has an end formula that is different from the negation of a definite, properly chosen formalized sentence.

The expressibility of a problem within the limits of certain means of expression, however, does not guarantee that its solution can be accomplished with these means as well. For the case of proof theory, it has indeed turned out that the finitist methods are not sufficient for the desired proofs of consistency of formalized theories.²⁸ \mid_{12} Therefore, it became necessary to extend the original finitist standpoint for proof theory to a “constructive” one, which can be characterized roughly by maintaining only the first two requirements of the three just mentioned ones, but dropping the third. Thus, we arrive at a position toward the methods in proof theory which is roughly the one that BROUWER’s intuitionism considers to be the only admissible one in mathematics in general.²⁹

²⁷[The objects which the general procedure (to be presented here) must accept as input arguments are those individual objects which may be given to the universal judgments on which the existential judgment depends, i.e. in whose scope the existential judgment occurs. See (Wirth, 2004; 2017) for several formal frameworks where this statement makes sense even in the absence of quantifiers.]

The conditions that these individual objects may be assumed to satisfy (according to “gewisse Bedingungen erfüllenden”/“satisfying certain conditions”) are those which arise from the logical context of the existential judgment, e.g. for an existential judgment in the conclusion of an implication, the individual objects may be assumed to satisfy the condition of the implication, &c. The required form of reasoning with this kind of logical context, called *hierarchical contextual reasoning*, was nicely formalized in (Autexier, 2003) for the first time.]

²⁸[Can this refer to anything else but (Gödel, 1931) ?]

²⁹A precise comparison is not to be demanded here, because the intuitionist position is actually not characterized by rules of application, but by a philosophical attitude. This is emphasized also by HEYTING, notwithstanding his formalization of intuitionist logic and mathematics. Moreover, a most relevant comparison of the methods of constructive proof theory with those of intuitionism is found in the more recent investigations of G. F. C. GRISS on negation-

Doch selbst bei dieser Erweiterung des finiten Standpunkts hat es nicht allenthalben sein Bewenden. Man sieht sich vielmehr bei der Behandlung gewisser Fragen dazu gedrängt, auch die zweite der obigen Forderungen fallen zu lassen. Dies ist zum Beispiel bei der Behandlung der Frage der Vollständigkeit des Systems der Regeln für die gewöhnliche Prädikatenlogik der Fall, die zuerst von KURT GÖDEL im positiven Sinne gelöst worden ist. Hier erfordert bereits die Formulierung des Ergebnisses, wenigstens wenn man sie in prägnanter und einfacher Form haben will, die Einführung einer nichtkonstruktiven Begriffsbildung. Eine finite Fassung des Ergebnisses und auch ein Beweis im finiten Rahmen läßt sich erzwingen, ist aber mit technischen Komplikationen belastet.

Dieses Beispiel des GÖDEL'schen Vollständigkeitssatzes ist zugleich dafür charakteristisch, daß die beweistheoretische Fragestellung in ihrer natürlichen Ausgestaltung nicht bei dem Problem verbleibt, welches aus der Kritik der üblichen Verfahren der klassischen Mathematik erwachsen ist. Auch HILBERT hat ja von vornherein die Aufgabe der Beweistheorie sehr weit gefaßt.³⁰ |₁₃

Unter den weitergehenden Fragestellungen sind nun auch etliche solche, bei denen die Verbindlichkeit der Anforderung einer methodischen Beschränkung als fraglich erscheint. So sind in neuerer Zeit verschiedenen³¹ erfolgreiche Untersuchungen, die in weiterem Sinne zum Felde der Beweistheorie gehören, im Rahmen der üblichen mathematischen Methodik, also ohne Beschränkung der Begriffsbildungen und Beweismethoden, durchgeführt worden. Andererseits haben verschiedene Autoren für jene speziellen beweistheoretischen Untersuchungen, die es mit den Fragen der Widerspruchsfreiheit zu tun haben, einen solchen Standpunkt gewählt, bei welchem von den drei vorhin formulierten Forderungen nur die erste zugrunde gelegt wird.

free intuitionist mathematics, where the handling of unreal (i.e. unsatisfiable) assumptions is avoided on principle (Proc. Kon. Ned. Adad. v. Wetensch. 49 (1946) [(Griss, 1946)], 53 (1950) [(Griss, 1950)], and 54 (1951) [(Griss, 1951a; 1951b; 1951c; 1951d)]).

³⁰Vgl. seine Äußerungen in dem Vortrag: Axiomatisches Denken (Math. Ann. 78, pp. 405–415 (1918)) [(Hilbert, 1918)].

³¹[Typo for “verschiedene”]

Even this extension of the finitist standpoint, however, does not suffice in all cases. In fact, for treating certain problems, we feel urged to drop the second of the above requirements as well. This is for example the case for the treatment of the problem of completeness of the rule system for ordinary predicate logic, first solved by KURT GÖDEL — in the positive sense. In this example, a concise and simple expression of the result already requires the introduction of a non-constructive concept formation. A finitist version of the result can be enforced, and also a proof by finitist means, but these versions are burdened with technical complication.³²

Thus, GÖDEL's completeness theorem provides also a characteristic example that proof-theoretic problem definitions in their natural embodiment already exceed the problem that resulted from the critique of the customary procedures in classical mathematics. Accordingly, HILBERT already gave a comprehensive outlook on the tasks of proof theory from the very beginning.³³ |₁₃

Moreover, for quite a few among the more advanced problem definitions, the commitment to a restriction of the methods may be questioned anyway. In fact, in more recent times, in the field of proof theory in the broader sense, various successful investigations have been realized with the full methodology of customary mathematics, restricting neither concept formations nor proof methods. Furthermore, even for proof-theoretic investigations actually concerned with the problems of consistency, various authors have chosen a standpoint that relies only on the first of the three previously stated restrictions.

³²[This refers to Herbrand's Fundamental Theorem (Herbrand, 1930), which, together with the Löwenheim–Skolem Theorem (Löwenheim, 1915), yields a finitist version of Gödel's completeness theorem (Gödel, 1930), cf. e.g. (Wirth & al., 2009; 2014), (Wirth, 2012; 2014).]

³³Cf. his statements in his talk: Axiomatisches Denken [Axiomatic Thought] (Math. Ann. 78, pp. 405–415 (1918)) [(Hilbert, 1918)].

Eine endgültige Entscheidung der Methodenfrage kann dieser Sachlage gegenüber jedenfalls nur erwartet werden, wenn man einen Überblick darüber hat, was die verschiedenen Methoden zu leisten vermögen. Es kann schwerlich behauptet werden, daß gegenwärtig eine Entscheidung jener Frage vorliegt, die nicht bloß durch eine vorgefaßte philosophische Ansicht bestimmt ist. Und bezüglich der verschiedenen sich bekämpfenden und heute üblichermaßen gegenübergestellten philosophischen Lehrmeinungen besteht der Verdacht, daß sie ungeklärte Voraussetzungen in sich schließen, die ihrerseits vielleicht eher fragwürdig sind als die angefochtenen mathematischen Theorien.

Angesichts dieser Sachlage erscheint es als das angemessene Verfahren, daß wir einerseits die methodische Richtlinie, die durch den Gesichtspunkt der finiten Betrachtung gegeben wird, im Auge behalten, andererseits uns [aber dadurch] nicht in der Methode festlegen. Dabei ist insbesondere der Umstand mitbestimmend, daß neuerdings die beweistheoretischen Untersuchungen in einen engeren Kontakt getreten sind mit den allgemeinen Theorien der abstrakten Algebra und Topologie, |₁₄ so daß die Aussicht sich eröffnet, daß die beweistheoretischen Methoden zu einem wirkungsvollen Hilfsmittel in diesen Gebieten sich entwickeln. Bei solchen Anwendungen fungiert die Beweistheorie nicht in der Rolle der Beweiskritik, sondern im Rahmen der üblichen Methoden des mathematischen Schließens, und es würde darum hier die Forderung der finiten Betrachtungsweise gar nicht am Platze sein.

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Zur Vorbereitung unserer beweistheoretischen Betrachtungen ist es nun auf jeden Fall wünschenswert, daß die spezifische Art der finiten Überlegung deutlich gemacht werde. Zur Illustrierung eignet sich besonders das Gebiet der elementaren Zahlentheorie, in welcher der Standpunkt der direkten inhaltlichen, ohne axiomatische Annahmen sich vollziehenden Überlegungen am reinsten ausgebildet ist. |₁₅

In any case — confronted with this overall situation — a final answer to the question which methods to choose can only be expected after attaining a comprehensive insight into the full capability of the different methods. It can hardly be claimed that an answer to that question has been given until today, unless determined by preconceived philosophical beliefs. And the various competing philosophical doctrines — nowadays mostly confronted with each other³⁴ — are suspected to comprise uncleared presuppositions. Moreover, these presuppositions may well be more questionable in turn than the mathematical theories under challenge.

In view of this situation, the appropriate procedure seems to be that we will keep track of the viewpoint of a finitist treatment as a guideline to determine our method of choice, but do not strictly limit ourselves to this method. A particular reason for this non-limitation is the fact that recently the proof-theoretic investigations have come into closer contact with the general theories of abstract algebra and topology,³⁵ [14] and thus the prospect is opening up that the proof-theoretic methods may develop into a powerful tool in these fields. In such applications, proof theory does not act in the rôle of a proof critique, but within the framework of the customary methods of mathematical inference, and therefore the demand for a finitist mode of consideration would not be appropriate here at all.

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For the preparation of our proof-theoretic considerations, it is now desirable in any case that the specific mode of finitist considerations is demonstrated. Particularly suitable for such an illustration is the field of elementary number theory, where the standpoint of direct contentual consideration without axiomatic assumptions is developed most purely. |15

³⁴[The English phrase “confronted with each other” is just as ambiguous as the German original “gegenübergestellt”, meaning either to be brought into opposition or to be arranged face-to-face for a comparison.]

³⁵[In the given context of the predicate calculus, this most probably refers to the PhD thesis (Hasenjaeger, 1950c) or to its improved version (Hasenjaeger, 1952b), both entitled “Topologische Untersuchungen zur Semantik und Syntax eines erweiterten Prädikatenkalküls” (“Topological Investigations on Semantics and Syntax of an Extended Predicate Calculus”). The papers (Stone, 1934) and (Mostowski, 1937; 1947) — cited and discussed in (Hasenjaeger, 1952b) — opened fascinating ways to transform logical problems into topological ones (with some effort), which then can be solved in an elegant and easy way by application of standard theorems of general topology. From these papers cited by Hasenjaeger, only (Mostowski, 1947) covers more than classical propositional logic. Thus, our typescript can hardly refer here to any publication previous to (Mostowski, 1947) and (Hasenjaeger, 1950c; 1952b). Indeed, we do not know of any previous publication to which this could refer w.r.t. the predicate calculus, and Hasenjaeger and Bernays did not know in 1952 of any such publication either: otherwise this would have been cited in (Hasenjaeger, 1952b), in particular as Bernays was referee of Hasenjaeger’s PhD thesis. All in all, this provides strong evidence that our typescript was not written before 1947, and hardly before 1950.]

With the previous paragraph ends our long excerpt from our typescript and its § 1, presented here together with its English translation on the right-hand sides (i.e. on the pages with uneven numbers).

2.5 Discussion and Excerpts of § 2 of Our Typescript

§ 2 of our typescript comes with the subsection headline “§ 2. Die elementare Zahlentheorie”³⁶ and contains a version similar to the first nine pages (pp. 20–28) of § 2 of (Hilbert & Bernays, 1934; 1968), which comes with a similar headline:

“§ 2. Die elementare Zahlentheorie. — Das finite Schließen und seine Grenzen.”³⁷

The first five paragraphs of § 2 of (Hilbert & Bernays, 1934; 1968), however, are missing in our typescript. For the first of these paragraphs, this is just a consequence of the omission of several paragraphs at the beginning of § 1 (cf. our § 2.4.1), because it is again on the subject of the “*existential form*”. The further four of these paragraphs missing in our typescript contain a digression into the subject of geometry.

2.5.1 A sentence following the second edition instead of the first one

A significant difference to (Hilbert & Bernays, 1934) is that — instead of the one-sentence paragraph

“Diese Figuren bilden eine Art von Ziffern; wir wollen hier das Wort ‚Ziffer‘ schlechtweg zur Bezeichnung *dieser* Figuren gebrauchen.”³⁸

of (Hilbert & Bernays, 1934, p. 21) — we find in our typescript the following sentence of (Hilbert & Bernays, 1968, p. 21):

“Wir wollen diese Figuren, mit einer leichten Abweichung vom gewohnten Sprachgebrauch, als ‚Ziffern‘ bezeichnen.”³⁹

It is unlikely that this text was just copied from the second edition, however, because our typescript contains the addition “(in Ermangelung eines besseren kurzen Ausdrucks)”⁴⁰, which is deleted by Bernays’ hand.

2.5.2 A significant improvement compared to both editions

Another significant difference in our typescript occurs in the enumerated list of five items found on page 21f. in both editions of Vol. I (Hilbert & Bernays, 1934; 1968). This list describes the syntactic form of the “Zeichen zur Mitteilung”⁴¹

³⁶“§ 2. Elementary Number Theory”

³⁷“§ 2. Elementary Number Theory. — Finitist Inference and its Limits.”

³⁸“These figures constitute a kind of numeral; and we will simply use the word ‘*numeral*’ to designate just *these* figures.”

³⁹“Deviating slightly from the common usage of language, we will call these figures ‘*numerals*’.”

⁴⁰“(lacking a better short expression)”

⁴¹“symbols for communication”

Item 1 of our typescript reads:

“kleine deutsche Buchstaben zur Bezeichnung für unbestimmt gelassene Ziffern;”

“small German letters for designating numerals that are left undetermined;”

Instead of this, we find

“Kleine deutsche Buchstaben zur Bezeichnung für irgendeine nicht festgelegte Ziffer;”

“Small German letters for designating an arbitrary, not determined numeral;”

in both editions of “Grundlagen der Mathematik”.

Together with other hints on the finitist standpoint found in our typescript, this clarification in our typescript was crucial for a change in translation in the third English edition (Hilbert & Bernays, 2017a), as compared to the first two English editions where we translated the latter German term — after communication with several members of the advisory board of the Hilbert–Bernays Project,⁴² who agreed that the meaning is ambiguous — with the similarly ambiguous term “an arbitrary indeterminate numeral”.⁴³ Our new translation in the third edition is:

“small German letters for designating arbitrary, not determined numerals;”

This new version clearly disambiguates these small German letters from *formal* variables as well as from arbitrary objects in the sense of Fine (1985). For a more detailed discussion see Note 21.6 on p. 21.b in the third English edition of “Grundlagen der Mathematik” (Hilbert & Bernays, 2017a).

In this context, it should be recalled that German letters — all over (Hilbert & Bernays, 1934; 1939; 1968; 1970) — do not denote *formal* variables (as Latin letters do). On the contrary, in the metalogical, finitist framework of Hilbert–Bernays’ “Grundlagen der Mathematik” — depending on the natural-language context — German letters may be used either for *free atoms* or for *free variables*

⁴²Cf. <https://w2.cs.uni-saarland.de/p/hilbertbernays/staff.html> or else <http://wirth.bplaced.net/p/hilbertbernays/staff.html>.

⁴³As long as we were not able to give the German term a unique reading, we had to be careful and use a translation that does not cut off any possibly meaningful reading. It was our typescript here that provided the information for such a unique reading. Now we can translate that reading and do not have to bother our readers with an ambiguous English term.

As long as there are several possible readings, a translator must not give only his favorite interpretation of the original in the translation. If the translator knows exactly what is meant, however, then that meaning should be captured in the translation as clearly and unambiguously as possible; otherwise the translator would propagate the weed of imperfect expression that always comes with the crop, in particular regarding the fruits of science.

in the terminology of (Wirth, 2017), which are both substantially different from the three kinds of *formal* variables of Hilbert–Bernays, i.e. from the bound individual variables, the free individual variables, and the free formula variables.

The technical term “*atom*” is from set theories (with atoms or urelements) and comes implicitly with a universal quantification,⁴⁴ whereas the term “*free variable*” is from free-variable semantic tableaux⁴⁵ and comes implicitly with an existential quantification.⁴⁶

Moreover, in (Wirth, 2004; 2017), these atoms and free variables were formalized and became part of the formal language, which also admits ε -restriction of the free variables by a choice condition. This means that the formal language offers a more general version of Hilbert’s epsilon — even without any quantification, neither quantification resulting in formulas (\forall, \exists) nor resulting in terms (ε, λ).

2.6 Discussion and Excerpts of § 3 of Our Typescript

§ 3 is the final section of our typescript. It comes with the subsection headline “§ 3. Überschreitung des finiten Standpunktes im mathematischen Schließen.”⁴⁷

2.6.1 From our typescript: the entire text of § 3, pp. 32–34

In its first two paragraphs of § 3, our typescript announces to discuss the most interesting and critical approach to finitism chosen in the penultimate paragraph of § 1 of our typescript. In the penultimate paragraphs on Pages 22 and 23 here, we have quoted the sentence on this choice and translated it as follows:

“In view of this situation, the appropriate procedure seems to be that we will keep track of the viewpoint of a finitist treatment as a guideline to determine our method of choice, but do not strictly limit ourselves to this method.”

The bad news is that § 3 is obviously truncated in our typescript: It breaks off abruptly after the first discussion of an abstract example on mathematical induction, not covering all what was announced in its first two paragraphs.

As this truncation results in a very short, but most interesting section (pp. 32–34), we present this section here in total, in the same way as we presented the new introduction from § 1 in § 2.4.3: The German original is found on the pages with even page numbers and our English translation on the respectively following pages with odd page numbers.

To this end, the remainder of the current page is left blank here.

⁴⁴Such as in the equations on p. 30 of (Hilbert & Bernays, 1934; 1968; 2017a).

⁴⁵*Sic!* Cf. (Fitting, 1990; 1996)

⁴⁶Such as in the equations on p. 28 of (Hilbert & Bernays, 1934; 1968; 2017a)

⁴⁷“§ 3. Transgression of the finitist standpoint in mathematical inference.”

Unsere ausgeführte Betrachtung der Anfangsgründe der Zahlentheorie diene dazu, uns das direkte inhaltliche, in Gedankenexperimenten an anschaulich vorgestellten Objekten sich vollziehende und von axiomatischen Annahmen freie Schließen in seiner Anwendung und Handhabung vorzuführen. Wir haben uns dabei an die methodische Einstellung gehalten, die wir anfangs nach HILBERT als den "finiten Standpunkt" bezeichnet haben. Wir wollen nun des näheren betrachten, wie man dazu veranlaßt wird, den finiten Standpunkt zu überschreiten. Dabei wollen wir anknüpfen an die früher gegebene Kennzeichnung des finiten Standpunktes, die ja mittels der drei charakteristischen Momente erfolgte: 1. Beschränkung der Gegenstände auf endliche diskrete Gebilde; 2. Beschränkung der Anwendung der logischen Formen des allgemeinen und des existentialen Urteils im Sinne der Vermeidung der Vorstellung von fertigen unendlichen Gesamtheiten; 3. Beschränkung der Annahmen auf solche über endliche Konfigurationen.

Diese Momente sind geordnet im Sinne einer zunehmenden Anforderung. Wir werden nun bei der Betrachtung der Überschreitung des finiten Standpunktes naturgemäß in entgegengesetzter Reihenfolge, im Sinne einer schrittweisen Abstreifung der Anforderungen, verfahren. |33

Ein Verstoß gegen die dritte Forderung, wonach alle Annahmen sich auf endliche Konfigurationen beziehen sollen, liegt bereits überall da vor, wo man die Annahme der Gültigkeit eines allgemeinen Satzes über Ziffern einführt.

Eine Veranlassung dazu ist insbesondere gegeben bei Anwendung der vollständigen Induktion zum Beweise von Sätzen, welche eine Beziehung $\mathfrak{A}(m, n)$ für beliebige Ziffern m, n behaupten. Soll die Induktion, etwa nach n , im finiten Sinne erfolgen, so muß bei dem Schluß von $\mathfrak{A}(m, n)$ auf $\mathfrak{A}(m, n+1)$ die Ziffer m festgehalten werden. In dieser Weise sind wir auch im vorigen Paragraphen bei den Beweisen der Rechengesetze für Summe und Produkt verfahren.

Häufig wird aber die vollständige Induktion so angewandt, daß man zunächst zeigt, daß für jede Ziffer m die Beziehung $\mathfrak{A}(m, 1)$ besteht, und sodann beweist, daß, falls für die Ziffer n bei jeder beliebigen Ziffer m $\mathfrak{A}(m, n)$ besteht, dann auch bei jeder Ziffer m $\mathfrak{A}(m, n+1)$ besteht. Man schließt daraus nach der vollständigen Induktion, daß für jede Ziffer n gilt, daß für jede Ziffer m $\mathfrak{A}(m, n+1)$ besteht.

Hier hat man in der zweiten zu beweisenden Behauptung einen Allsatz als Prämisse; es wird ja angenommen, daß (für den fixierten Wert n) bei jeder Ziffer m die Beziehung $\mathfrak{A}(m, n)$ bestehe. Dieses Vorausgesetzte können wir uns nicht in der Vorstellung eigentlich vergegenwärtigen.

Our treatment of the basics of number theory was meant to demonstrate the application and the use of direct contentual inference in thought experiments on intuitively conceived objects, free of axiomatic assumptions.⁴⁸ In this treatment, we have observed the methodological attitude we initially called the “finitist standpoint” according to HILBERT. We now want to discuss in more detail what may be the cause for a transgression of the finitist standpoint. We want to follow here the previously given characterization of the finitist standpoint by means of three characteristic moments: 1. limitation of any object to be a finite discrete entity; 2. limitation of any application of the logical forms of the universal and the existential judgment to avoid the conception of any completed infinite totality; 3. limitation of any assumption to refer only to finite configurations.

These moments are ordered by increasing demand. Now, in our discussion of the transgression of the finitist standpoint — in the sense of a stepwise discarding of demands — we will naturally proceed in the reverse order. |33

A violation of the third requirement, according to which all assumptions should refer to finite configurations, is already given wherever one introduces the assumption of the validity of a universal sentence about numerals.

In particular, a cause for such a violation may be given in an application of mathematical induction for the proof of a sentence that asserts a relation $\mathfrak{A}(m, n)$ for arbitrary numerals m, n . If the induction, say on n , is to be carried out in the finitist sense, then the numeral m must be held constant in the conclusion step from $\mathfrak{A}(m, n)$ to $\mathfrak{A}(m, n+1)$. In this way we proceeded also in the previous section in the proofs of the laws of calculation for sum and product.

Frequently, however, mathematical induction is applied in such a way that one first shows that the relation $\mathfrak{A}(m, 1)$ holds for every numeral m , and then proves for the numeral n that $\mathfrak{A}(m, n+1)$ holds for every numeral m , under the assumption that $\mathfrak{A}(m, n)$ holds for any numeral m . From this, we may conclude by mathematical induction for every numeral n : $\mathfrak{A}(m, n+1)$ holds for every numeral m .

In the second assertion to be shown here, we have a universal sentence as premise; indeed, we assume that (for the fixed value n) the relation $\mathfrak{A}(m, n)$ holds for every numeral m . In our conception, we are not really able to bring clearly to our mind what we have to presuppose here.⁴⁹

⁴⁸[This sentence is very similar to the first sentence of the third paragraph on p. 32 in (Hilbert & Bernays, 1934; 1968; 2017a).]

⁴⁹[Indeed, we cannot visualize — element by element — the set $\{ \mathfrak{A}(\mathfrak{z}, n) \mid \mathfrak{z} \in \mathbf{N} \}$ of all potential assumptions, but only effectively given, finite subsets of it.]

Freilich läßt sich in vielen Fällen die genannte Form der Anwendung der vollständigen Induktion vom finiten Standpunkt motivieren. Das ist z. B. dann der Fall, wenn beim Beweis des Bestehens von $\mathfrak{A}(m, n+1)$ für ein bestimmtes m die Voraussetzung des Bestehens von $\mathfrak{A}(\mathfrak{s}, n)$ für beliebige \mathfrak{s} nur $|_{34}$ in solcher Weise zur Anwendung kommt, daß eine durch m und n bestimmte endliche Anzahl von Beziehungen

$$(1) \quad \mathfrak{A}(f_1, n), \quad \dots, \quad \mathfrak{A}(f_r, n)$$

benutzt wird, worin f_1, \dots, f_r gewisse aus m und n zu ermittelnde Ziffern sind.

In diesem Falle kommt ja der Beweis des hypothetischen Satzes mit der Allprämisse darauf hinaus, daß man zeigt, daß sich die Feststellung der Beziehung $\mathfrak{A}(m, n+1)$ (für die fixierten Ziffern m, n) zurückführen läßt auf die Feststellung der Beziehungen (1), und damit ist ein Regreß gegeben, der die Feststellung von $\mathfrak{A}(m, n+1)$ in einer begrenzten Zahl von Schritten auf die Feststellung von Beziehungen

$$\mathfrak{A}(\mathfrak{s}_1, 1), \quad \dots, \quad \mathfrak{A}(\mathfrak{s}_s, 1)$$

zurückführt, und für diese wird durch den anfänglichen Beweis des Bestehens von $\mathfrak{A}(m, 1)$ für beliebige m das allgemeine Verfahren gegeben.

Diese Art der Rechtfertigung der erweiterten Form der vollständigen Induktion mit Allsätzen als Prämissen ist jedoch nicht generell anwendbar. Insbesondere erwächst eine Schwierigkeit aus dem Umstand, daß derartige erweiterte Induktionen in komplizierter Weise ineinandergeschachtelt sein können. Wir wollen einen typischen Fall dieser Art näher betrachten. Es handelt sich dabei um einen konstruktiv behandelbaren Teil der CANTORSchen Theorie der transfiniten Ordinalzahlen. $|_{35}$

Nevertheless, the considered application of mathematical induction can be justified from the finitist standpoint in many cases.⁵⁰ For instance, such a justification is always possible if — in the proof that $\mathfrak{A}(m, n+1)$ holds for a particular m — the application of the hypothesis that $\mathfrak{A}(\mathfrak{z}, n)$ holds for arbitrary \mathfrak{z} can be restricted |₃₄ to a finite number of [instances of this universal sentence in the form of] relations

$$(1) \quad \mathfrak{A}(f_1, n), \quad \dots, \quad \mathfrak{A}(f_r, n),$$

where [r as well as] f_1, \dots, f_r are certain numerals that have to be determined from m and n .

In this case the proof of the proposition⁵¹ with the universal premise amounts to showing that the verification of the relation $\mathfrak{A}(m, n+1)$ (for the fixed numerals m, n) can be reduced to the verification of the relations (1). And therefore, we are then given a regression procedure that reduces the verification of $\mathfrak{A}(m, n+1)$ to the verification of relations

$$\mathfrak{A}(\mathfrak{z}_1, 1), \quad \dots, \quad \mathfrak{A}(\mathfrak{z}_s, 1)$$

in a limited number of steps. Moreover, for these relations, there is a general procedure given by the initial proof that $\mathfrak{A}(m, 1)$ holds for arbitrary m .

This way of justifying the extended form of mathematical induction with universal sentences as premises, however, is not applicable in general. In particular, one of the difficulties that may arise here is the circumstance that such extended inductions can be nested into each other in a complicated way. Let us discuss a typical case of this kind in more detail. We will deal here with a part of CANTOR's theory of transfinite ordinal numbers that can be treated constructively. |₃₅

⁵⁰[For a formal treatment of this motivation see pp. 348–351 of (Hilbert & Bernays, 1968; 2017c).]

⁵¹[The occurrence of “hypothetischen” in the German original is to make clear that “Satzes” (“sentence”) does not refer to a theorem, but just to a proposition, namely the induction step here. The German wording clashes with the standard notion of an *induction hypothesis*. Indeed, we cannot translate this occurrence as “hypothetical sentence” here because we already translated “Voraussetzung” (“presupposition”) in the previous paragraph as “hypothesis”.]

With the previous paragraph ends our excerpt presenting § 3 of our typescript in total — here together with its English translation on the right-hand sides (i.e. on the pages with uneven numbers).

The original text suddenly breaks off here. It is not clear whether the remainder is just incorrectly filed or whether Bernays further draft (in GABELSBERGER shorthand?) — which must have been written before the introduction to this subsection — was never put into typewriting.

3 Trying to Find Hints on the Time of Writing

3.1 § 1 of Our Typescript (Hints: Before 1929, 1931, or 1951–1977)

3.1.1 Possible explanations for the missing of the two paragraphs on the “existential form” in the introductory part

To explain why the two paragraphs from the beginning of § 1 of (Hilbert & Bernays, 1934; 1968) are missing in our typescript (as described in § 2.4.1), we see the following *three options*:

1. The missing of the two paragraphs may indicate that our typescript was written before 1929 for the following reason: Soon later the subject of these paragraphs, the “existential form”, became an integral part of Bernays’ standard introduction, never omitted in any publication on the foundations of mathematics; cf. the initial sections of (Bernays, 1930/31), (Hilbert & Bernays, 1934; 1968).
2. It could have been the case that Bernays or the Hilbert school in logic temporarily dropped the idea of the “existential Form” after the shock of Gödel’s incompleteness theorems, say in 1931.
3. A third option is that the whole text was written by Gisbert Hasenjaeger during his stay as Bernays’ assistant in Zürich, or later as a consequence of this stay. What speaks for this option is the overall improved pedagogical quality of our typescript as compared to the corresponding sections in (Hilbert & Bernays, 1934; 1968). Then these paragraphs may be not actually missing, but just deferred to a later section where they are more appropriate from the pedagogical viewpoint.

3.1.2 The completely unknown introduction

In the completely unknown introduction to the foundations of mathematics and Hilbert’s proof theory (cf. § 2.4.3), there are some references to relevant published work that may help to date our typescript.

1. There is the following reference (cf. Note 28, § 2.4.3):

“Tatsächlich hat es sich im Fall der Beweistheorie herausgestellt, daß für die gewünschten Nachweise der Widerspruchsfreiheit formalisierter Theorien die finiten Methoden nicht zulänglich sind.”

“For the case of proof theory, it has indeed turned out that the finitist methods are not sufficient for the desired proofs of consistency of formalized theories.”

We do not think that this could have been written before autumn 1930, when Gödel’s incompleteness theorems (1931) became known. If this is so, then

our typescript cannot have been written before autumn 1930, which excludes the first option of §3.1.1.

2. Moreover, there is the reference to (Griss, 1951a; 1951b; 1951c; 1951d) in Notes 26 and 29 of §2.4.3. Unless the notes on the extra pages (cf. §2.2) were added much later to our typescript than our typescript itself was originally written (which is very unlikely because the position markers of the footnotes are already part of our typescript), this means that our typescript cannot have been written before 1951.
3. Furthermore, there is also the following reference (cf. Note 35, §2.4.3):

“Dabei ist insbesondere der Umstand mitbestimmend, daß neuerdings die beweistheoretischen Untersuchungen in einen engeren Kontakt getreten sind mit den allgemeinen Theorien der abstrakten Algebra und Topologie, |₁₄ so daß die Aussicht sich eröffnet, daß die beweistheoretischen Methoden zu einem wirkungsvollen Hilfsmittel in diesen Gebieten sich entwickeln.”

“A particular reason for this non-limitation is the fact that recently the proof-theoretic investigations have come into closer contact with the general theories of abstract algebra and topology, |₁₄ and thus the prospect is opening up that the proof-theoretic methods may develop into a powerful tool in these fields.”

As we have discussed in detail in Note 35, there is strong evidence that this was not written before 1947, and hardly before 1950.

If our assessment in the last two items is correct, then also the second option of §3.1.1 can be excluded and our typescript was probably written by Hasenjaeger during or after his stay with Bernays in Zürich.

3.2 §2 of Our Typescript (Hints: Hasenjaeger, 1951–1977)

The sentence of our typescript found in the second instead of the first edition of “Grundlagen der Mathematik” (which we discussed in §2.5.1) clearly speaks in favor of a version written during the preparation of the second edition; and the only work in this context we know about is the joint work of Hasenjaeger and Bernays.

The sentence of our typescript improving on both editions of “Grundlagen der Mathematik” (which we discussed in §2.5.2) makes it likely that it was written with the intention of a thorough revision; and the only such attempt we know about is the joint work of Hasenjaeger and Bernays. Moreover, as Bernays used to keep track of any of his own corrections very carefully in his author’s copies,⁵²

⁵²Such as the one of the first edition (Hilbert & Bernays, 1934; 1939) owned by Erwin Engeler and the one of the second edition (Hilbert & Bernays, 1968; 1970) owned by René Bernays, cf. (Hilbert & Bernays, 2017a; 2017b; 2017c).

where we do not find this correction, however, this improvement over both editions also indicates that our typescript was the result of a collaboration indeed.

3.3 § 3 of Our Typescript (No Hints)

Although § 3 of our typescript is very interesting and we would be keen on reading the remainder of it if it were found, it breaks off too soon for giving us a clear hint on the time of its writing.

4 Hasenjaeger and Bernays

As we have seen, for finding out the time of writing of our typescript, it may be helpful to find out more about Hasenjaeger's scholarship as Bernays' assistant in Zürich for the preparation of the second edition of "Grundlagen der Mathematik" in the early 1950s (cf. § 1.2.2). So the questions is: *What do we know about the relation of Hasenjaeger and Bernays around the year 1950?*

Besides several biographical remarks from Hasenjaeger in (Menzler-Trott, 2001), there seem to be essentially only two non-trivial biographical texts on Hasenjaeger: One is a laudation by Diller (2000); the other one is on his time as a cryptologist responsible for the security of the German Enigma in the Second World War and appears in similar forms in (Schmeh, 2005; 2009; 2013). There we learn that he was born June 1, 1919, in Hildesheim (Germany) as a son of a lawyer;⁵³ that he was seriously wounded on January 2, 1942, as a German soldier in Russia, and that the famous logician, philosopher, and theologian Heinrich Scholz (1884–1956) saved him from being ordered to the Russian front again, by recruiting him for the cryptology department of the High Command of the German Armed Forces (OKW/Chi) in Berlin.

As we were neither able to find a reasonable short CV of Hasenjaeger in publications nor in the WWW, we have put his own CV together with an English translation and some further remarks into our Appendix. For a list of his publications, see our hint at the beginning of our Bibliography.

4.1 Early Relation

Hasenjaeger and Bernays were exchanging letters on the first edition of "Grundlagen der Mathematik" since 1943, and from 1949 on also on Hasenjaeger's own work.⁵⁴ They definitely met each other in autumn 1949 at the "Kolloquium zur

⁵³According to (Menzler-Trott, 2001, p.186) and also to Hasenjaeger's daughter Beate Becker, Gisbert Hasenjaeger's father was Edwin Hasenjaeger (1888–1972), who was the mayor of Mülheim an der Ruhr (Germany) from 1936 to 1946, and a very close friend of Heinrich Scholz (according to https://www.muelheim-ruhr.de/cms/edwin_hasenjaeger_-_portrait_eines_oberbuergemeisters_1936-19461.html).

⁵⁴In particular, Hasenjaeger and Bernays were exchanging letters on the following publications: (Hasenjaeger, 1950a; 1950b; 1950c; 1952a; 1953).

Logistik und der mathematischen Grundlagenforschung”, Sept. 27 – Oct. 1, 1949, Mathematisches Forschungsinstitut Oberwolfach (MFO), Oberwolfach (Germany). Indeed, the following photo⁵⁵ shows Hasenjaeger in the back row to the right behind Bernays (with H. Arnold Schmidt in dark suit to the right of Bernays, and Kurt Schütte rightmost):



Bernays seems to have reviewed the PhD thesis of Hasenjaeger (1950c) before May 24, 1950, which is the date of the final report by Hasenjaeger’s supervisor Heinrich Scholz.⁵⁶

4.2 Joint Work on “Grundlagen der Mathematik”

All in all, three scholarships were granted to Hasenjaeger by the ETH Board,⁵⁷ with the goal to assist Bernays in the preparation of the second edition of “Grundlagen der Mathematik”. Hasenjaeger’s first stay as Bernays’ assistant in Zürich took place in winter term 1950/51.⁵⁸ As planned from the beginning, a second stay followed in summer term 1951.

Only after this second stay in Zürich, however, Hasenjaeger produced the *first proper sketch*⁵⁹ of the overall layout for the second edition of “Grundlagen der Mathematik”, in the form of a rough table of contents. This sketch was and still is attached to Hasenjaeger’s letter (1951) to Bernays. It is an elaborated and augmented protocol of the first detailed discussion with Bernays on the subject of the layout for the second edition of Hilbert–Bernays, which probably took place

⁵⁵Source: Univ. Archiv Freiburg (Breisgau, Germany): Depositbestand E6. Scanned, imprinted, and usage permitted by Mathematisches Forschungsinstitut Oberwolfach (MFO), Oberwolfach (Germany): https://opc.mfo.de/detail?photo_id=11199

⁵⁶For more information on Hasenjaeger’ promotion, see the item labeled (Hasenjaeger, 1950c) in our Bibliography.

⁵⁷In German: “ETH-Rat”; at that time called “Schweizerischer Schulrat”. For the grants see (Hasenjaeger, 1951; 1952g).

⁵⁸Cf. (Hasenjaeger, 1950e).

in Zürich at the very end of the summer term 1951.⁶⁰

During Hasenjaeger's first two semesters in Zürich, there were neither publications under his name nor work on the "Grundlagen der Mathematik". Therefore, a further stay of Hasenjaeger as Bernays' assistant was strongly suggested by Heinrich Scholz already February 1, 1951, in Scholz' letter (1951b) to Hasenjaeger. This third stay indeed took place during winter term 1951/52 at the ETH Zurich, and Hasenjaeger left Zürich soon after February 18, 1952.⁶¹

Most surprisingly, however, this first proper sketch attached to (Hasenjaeger, 1951) *does not show the slightest similarity with our typescript here*. As the non-matching of this sketch (handwritten by Hasenjaeger) with our typescript is crucial for the timing assessment of our typescript, let us quote the "Introduction" ("Einleitung") right at the beginning of it, roughly with the original line breaking:

“Die logische Struktur der mathematischen Theorien.

Gliederungsentwurf.

Einleitung (E)

- a) Endliche Bereiche. Das finite Schliessen in Bezug auf "offene" Gesamtheiten (Spezies). Endliche Zahlen – Ordnungszahlen. Unendliche Bereiche. Beweise als mathematische Objekte.
- b) Elementare Verbands- und/oder Gruppen-theorie. Dabei Aufsuchen der einfachsten Schlussweisen. Aufstellung eines provisorischen Aussagen-Kalküls (evtl. Dingvariablen zunächst nur als Mitteilungszeichen; dann Überlegung zum) elementaren Kalkül mit freien Variablen.

⁵⁹This sketch attached to (Hasenjaeger, 1951) may only have been written in response to Scholz' letter (1951a) to Bernays, where Scholz expresses his worries raised by an oral report from Hasenjaeger (who must have met Scholz in Münster between the years 1950 and 1951) that the work on "Grundlagen der Mathematik" had not even started yet:

"Inzwischen ist Herr Hasenjaeger hier erschienen, und ich habe ihn als allererstes gefragt nach dem Stande der Arbeit an dem Grundlagenbuch. Er hat mir gesagt, dass diese Arbeit noch gar nicht hat anlaufen können, weil Sie die Mengenlehre erst fertig machen müssen. Dies hat mich nun wirklich so erschreckt, dass ich es Ihnen mit der ersten Möglichkeit sagen muss."

In English: "In the meantime, Mr. Hasenjaeger appeared here, and my very first question was about the state of affairs of the work on the foundations book. He told me that this work could not even have started yet, because you first have to complete the set theory. Now this has alarmed me to such an extent that I have to tell this to you with the first opportunity."

The noun phrase "the set theory" refers to (Bernays, 1954) (received March 30, 1953), where Hasenjaeger is mentioned only in the form of a reference to (Hasenjaeger, 1953).

In the eyes of SCHOLZ, it was even worse for Hasenjaeger's career that the work of Hasenjaeger and Bernays on the second edition did not even start before summer 1951, cf. (Scholz, 1951c; 1951d).

⁶⁰For estimating the time of the detailed discussion with Bernays, see also (Scholz, 1951d) in addition to (Hasenjaeger, 1951). In the letter (Hasenjaeger, 1951), a very rough previous sketch is mentioned to have been brought into accordance with the elaborated sketch attached to (Hasenjaeger, 1951). This very rough sketch is probably a protocol of a very first short discussion with Bernays on the Hilbert–Bernays subject, which took place at the end of winter term 1950/51 in Zürich.

⁶¹Cf. (Hasenjaeger, 1952d; 1970).

- c) Kombinatorischer Reichtum der “Beweistheorie”. Einbettung der Beweistheorie in die Zahlentheorie (Arithmetisierung). ? Elementare Probleme, die nichtelementare Methoden erfordern; Vergleich mit Zahlentheorie.
- d) Widerspruchsfreiheit. Behandlung durch endliche Modelle und durch Entscheidungsverfahren. Entscheidungsverfahren an sich (Beisp. Verb.theorie).”

English translation:

“The logical structure of mathematical theories.

Layout outline.

Introduction (E)

- a) Finite domains. Finitist inference w.r.t. “open” totalities (species). Finite numbers — ordinal numbers. Infinite domains. Proofs as mathematical objects.
- b) Elementary lattice and/or group theory. Thereby exploration of the most simple modes of inference. Establishing a provisional propositional calculus (maybe object variables first only as symbols for communication; then considerations on the) elementary calculus with free variables.
- c) Combinatorial abundance of “proof theory”. Embedding of proof theory into number theory (arithmetization). ? Elementary problems that require non-elementary methods; comparison to number theory.
- d) Consistency. Treatment by means of finite models and decision procedures. Decision procedures themselves (example lattice theory).”

All in all, the complete sketch attached to (Hasenjaeger, 1951) shows that the goal of the joint work of Hasenjaeger and Bernays was *a complete rewriting or even a completely new writing of the book right from the beginning* — definitely not just an integration of new parts into the first edition as found in our typescript and as indicated in the “Preface to the Second Edition”, cf. § 1.2.2.

From the sequence of letters and postcards (Hasenjaeger, 1952e), (Bernays, 1952), (Hasenjaeger, 1952f), it becomes clear that the project of a complete reorganization of the book was still continued in 1952; in particular a completely new version of the treatment of the ι -operator is discussed in this sequence of letters. Moreover, Hasenjaeger’s state-of-the-art introduction of the ι -operator (1952h) — ready for press and fundamentally different from the obsolete treatment in both Hilbert–Bernays editions — was probably already attached to the first letter (Hasenjaeger, 1952e). Note that, according to the sketch attached to (Hasenjaeger, 1951), the ι was to be treated in Chapters VI and VII of all in all 12 chapters (incl. the unnumbered introduction and the supplement chapters).

All in all, *Bernays may give us a wrong impression* by using the subjunctive “erfordert hätte” (“would have required”) in the “Preface to the Second Edition”, cf. § 1.2.2. Indeed, as a complete rewriting of the book was obviously planned

from the very beginning of the work in summer 1951 and pursued far beyond the end of Hasenjaeger's last stay in Zürich in February 1952, it cannot be the case that the planned work was *reduced* when — “already back then” when “Mr. G. HASENJAEGER came to Zürich for some time” — “it became obvious that the integration of the many new results in the area of proof theory would have required a complete reorganization of the book.” Of course, Bernays does not explicitly state that the work on a complete reorganization was *reduced* “already back then”, but his directly following sentence already speaks only of the *reduced* version of the second edition — without mentioning *any other point in time* for the decision on this reduction, cf. § 1.2.2. Considering Bernays most contextual style of writing, where a sentence often can only be understood correctly if the reader has the meaning of several sentences of the context completely in mind, we never had the slightest doubt on the identity of these two points in time for several decades, until we started our investigations on the joint work of Hasenjaeger and Bernays.

The truth about the joint work of Hasenjaeger and Bernays on the “Grundlagen der Mathematik” seems to be that this work was given up only after a completely reorganized version for the first volume was already written, including *two* completely new chapters on the ι -operator (among other subjects). Be reminded that the ι -operator is treated in both published editions at the very end of the first volume only in a *single* chapter, which does not even meet the state of the art of the original treatment of the ε in the second volume of the first edition (Hilbert & Bernays, 1939).

In addition to this huge work on the entire first volume, the joint work of Hasenjaeger and Bernays also includes the writing of our typescript (not at all following the “Introduction” presented above) and, moreover, at least the still lost continuation of its § 3.

5 Conclusion

5.1 Overall Assessment of Our Typescript

In spite of some minor evidence that our typescript was written before 1929 (cf. § 3.1.1(1)) or in the year 1931 (cf. § 3.1.1(2)), the overwhelming evidence says that it was written not before 1951 (as explicated in §§ 3.1.2 and 3.2). In § 4.2, we then studied the joint work of Bernays and Hasenjaeger in Zürich 1950–1952, where we found some further evidence that our typescript resulted from that joint work. After all, the strongest evidence we have is the following: Two carbon copies of our typescript were found in Hasenjaeger's legacy — together with the only source for the footnotes to our typescript! As Hasenjaeger seems to have added these footnotes to a typescript — corrected by BERNAYS' hand and with marks indicating the places of the anchors for these footnotes — Hasenjaeger must have been involved in the production of our typescript — and, for this and several other reasons (cf. §§ 2.1 and 2.2), he is most probably the actual typist of it as well.

What we still do not know, however, is the time of writing, and we cannot exclude any year from 1951 to 1968 (when the second edition of the first volume of “Grundlagen der Mathematik” was published). Because of the improvements of our typescript over both editions of Hilbert–Bernays (cf. § 2.5.2, § 3.1.1(3), § 3.2), even the years from 1968 to 1977 (when Bernays died) cannot be excluded. Most likely, however, is the time of the winter term 1951/52, the last one Hasenjaeger stayed with Bernays at the ETH in Zürich (as explicated in § 4.2).

In any case, the finding of this incorrectly filed typescript is essential in the context of Hilbert and Bernays’ “Grundlagen der Mathematik”, because it provides us with some new insights⁶² into the finitist standpoint and its development, in particular in connection with the two editions of the *first volume* of “Grundlagen der Mathematik” (Hilbert & Bernays, 1934; 1968).

5.2 Three Most Interesting Scripts Still Missing

As we do not know any way to find the following three scripts with our limited resources, the treasure quest remains open for future prospectors:

1. It is worthwhile to invest further effort and to search the legacy of Bernays and the ETH archives more broadly: There may be a chance to find an incorrectly filed continuation of our typescript, at least the remainder of its § 3, possibly in form of a draft partly in Bernays’ Gabelsberger shorthand.

Such a finding may change our point of view on Bernays’ ideas on the foundations of mathematics considerably.

2. Be aware that our typescript must not be mistaken for the *other script* that Hasenjaeger wrote according to his handwritten sketch attached to (Hasenjaeger, 1951) — probably a typescript of a completely rewritten first volume of “Grundlagen der Mathematik”, cf. § 4.2.

This other script would be a real bonanza regarding the views of Bernays on the foundations of mathematics in the early 1950s.

We have no idea on the whereabouts of this other script.

It would be a real pity if this major work of Hasenjaeger and Bernays, which can hardly be overlooked in any library by its mere size, really remained lost.

3. Finally, to find the initial, nearly completed script for the first edition remains, of course, one of the biggest wishes of maybe every historian of modern logic.

⁶²For instance, § 2 of our typescript has provided us with some new insight into the finitist standpoint, which helped us to improve the translation of the first part of § 2 of (Hilbert & Bernays, 1934; 1968) in (Hilbert & Bernays, 2017a) considerably, cf. § 2.5.2.

Moreover — and maybe more important — also § 3 of our typescript has provided us with some new insight into the finitist standpoint, which led to Note 349.2 in (Hilbert & Bernays, 2017c, p. 349).

Curriculum Vitae of Gisbert Hasenjaeger by Himself

German Original

The following is a reprint of (Hasenjaeger, 1997), written by Gisbert Hasenjaeger himself. Also the layout is exactly the one of Hasenjaeger's ASCII file.

Erstellt am: 16.08.1997

VITA Gisbert HASENJAEGER, geb. 01.06.19 Hildesheim

[REM mit Korrekturen im Sinne von Standardisierung und von Kürzungen einverstanden; Mitteilung des Ergebnisses erbeten]

1937 Abitur in Mülheim a.d. Ruhr. 1937-1939 Arbeits- und Wehrdienst.

1942 Nach "Kopfschuß" (2.1.) und Rekonvalensenz bei OKW/CHI gegen (wie ich viel später erfuhr) A. Turing eingesetzt.

1945-1950 Studium in Münster/Westf., dabei seit Beginn (wissensch.) Hilfskraft am Seminar/Institut für Mathematische Logik und Grundlagenforschung.

1950 Promotion zum Dr.rer.nat. bei Heinrich Scholz mit der Dissertation "Topologische Untersuchungen zur Semantik und Syntax eines erweiterten Prädikatenkalküls".

1950 wissenschaftlicher Assistent am o.a. Institut.

1950/51 (das akademische Jahr) Stipendiat Gast bei Paul Bernays a.d. ETH Zürich.

1953 Habilitation und *venia legendi* (Mathematische Logik und Grundlagenforschung) an der Universität Münster (Habilitationsschrift. Widerspruchsfreie Axiomensysteme ohne Standard-Modell).

1955 Diätendozentur, 1960 apl. Professur U Münster.

1961 SS und WS 61/62 Vertretung einer neu gegründeten Professur für Logik an der Philosophischen Fakultät der Universität Bonn.

1962 Berufung auf diese (a.o.) Professur, für Logik und Grundlagenforschung. [REM Durch das Fehlen des Adjektivs (math.) und den heutigen Sprachgebrauch ist das nun wohl "ein weites Feld", welches zu beackern ich mir nicht mehr zumuten möchte.]

1964 Persönlicher Ordinarius, 1966 o. Prof.

1964/5 (das akademische Jahr) Gast am Institut for Advanced Studies Princeton N.J.

1970/1 (das akademische Jahr) Gast-Professur an der University of Illinois in Urbana/Champaign.

1984 Emeritierung U Bonn.

English translation

Date of Generation: Aug. 16, 1997

VITA Gisbert HASENJAEGER, born June 1, 1919, in Hildesheim (Germany)

[REM[ark?:] I agree with corrections in the sense of standardization and abbreviation, but ask for communication of the result]

1937 A-levels (Abitur) in Mülheim a.d. Ruhr.

1937–1939 Working and Military Service.

1942 After shot in the head (Jan. 2) and reconvalescence at OKW/CHI⁶³ mission against A. Turing (as I was told much later).

1945–1950 Studies in Münster/Westf., from the beginning Scientific Assistant at the Seminar/Institute for Mathematical Logic and Foundational Research

1950 Promotion to Dr. rer. nat. by Heinrich Scholz with the dissertation “Topological Investigations on Semantics and Syntax of an Extended Predicate Calculus”.

1950 Postdoctoral Research Assistant at the above-mentioned institute.

1950/51 (the academic year) Scholarship Awardee as a guest of Paul Bernays at the ETH Zurich.

1953 Habilitation and *venia legendi* (mathematical logic and foundational research) at the Univ. Münster

(habilitation thesis: “Consistent Axiom Systems Without Standard Model”).

1955 Position of Supernumerary University Lecturer, 1960 Adjunct Professor, Univ. Münster

1961/62 (summer and winter term) Deputy for a newly founded logic professorship at the philosophical faculty at Univ. Bonn.

1962 Appointed to this professorship, for logic and foundational research.

[REM As the adjective “math.” is lacking here and according to today’s usage of language, this seems to be a “wide field” nowadays, which I would definitely not want to treat anymore.]

1964 Personal Professor (“*Persönlicher Ordinarius*”),

1966 Full Professor (“*Ordinarius*”).

1964/5 (the academic year) Guest at the Institut for Advanced Studies Princeton N.J.

1970/1 (the academic year) Guest Professorship at the Univ. of Illinois in Urbana/Champaign.

1984 Professor Emeritus, Univ. Bonn.

⁶³Actually: “OKW/Chi”, i.e. *Oberkommando der Wehrmacht/Chiffrierabteilung*.

Remarks on Hasenjaeger's Curriculum Vitae

To provide some more detail and to overcome errors in other publications, we would like to add here three remarks to Hasenjaeger's Curriculum Vitae.⁶⁴

First we would like to clarify some difficult point regarding his stay with Bernays in Zürich: In his CV (Hasenjaeger, 1997), Hasenjaeger omits the last semester of his grants in Zürich. In his draft for an English CV (Hasenjaeger, 1970), however, he writes: "From autumn 1950 to spring 1952 I worked under a scholarship with P. Bernays at the ETH, Switzerland." Up to now, we can only be certain that Hasenjaeger was in Zürich for the first 18 days of February 1952, i.e. during the very end of the winter term 1951/52, and that he planned to meet Scholz in Münster (Westfalen) not before Feb. 29, 1952. The hard evidence for this is the letter (Hasenjaeger, 1952d) to Scholz, written on Feb. 18, 1952, in Zürich.

Second, we would like to remark that — in spite of his severe war injury in 1942, as an artillerist in the German attack of Russia, after he had been drafted for military service in 1939 — he was able to keep a good work/life balance: In 1956 he married Irmhild Reinländer, a former student assistant of Heinrich Scholz and a mathematics and physics teacher in Soest (Germany) (1955–1957). Moreover, he had three children with his wife: Andreas (*1957), Beate (*1959), and Cordula (*1961). When he was Visitor at the Institute for Advanced Study, Princeton (NJ), in winter term 1964/65 and summer term 1965, or Visiting Professor at the Univ. of Illinois at Urbana–Champaign in winter term 1970/71 and summer term 1971, he took his whole family with himself to the US. Gisbert Hasenjaeger died on the estate of his wife in Plettenberg (Germany), Sept. 2, 2006. She died as his widow in 2012.

And a final, but lengthy remark has become necessary to clarify some important detail on his working and military service (Reichsarbeits- und Wehrdienst) during the years 1937–1939, namely that he did *not volunteer* for this — not only because this was not possible at that time, but also because neither he nor his family were in any way admirers the Nazi regime. Moreover, Gisbert Hasenjaeger himself hated anything military. Contrary to a text that wrongly puts Hasenjaeger close to being an admirer of the Nazi reign of terror, found on Dec. 28, 2019, in the German, English and French Wikipedia entries on Gisbert Hasenjaeger and also in the many places that automatically copy this attack to the soundness of public knowledge, Hasenjaeger did *not* do these services *voluntarily*. For instance, in the German version we could read: "Hasenjaeger war danach freiwillig beim Reichsarbeitsdienst und leistete seinen Wehrdienst." By Nov. 11, 2021, after many corrections by us and others, this nonsense has reappeared again in the English and French Wiki-

⁶⁴Sources: Correspondence Hasenjaeger–Bernays: (Hasenjaeger, 1950d; 1950e; 1952e; 1952f). Three Curricula Vitae by Hasenjaeger himself: (Hasenjaeger, 1952g; 1970; 1997). Published articles: (Schmeh, 2005; 2009; 2013). Unpublished laudation: (Diller, 2000). Communication with Gisbert Hasenjaeger's daughter Beate Becker by E-mail and by phone, Jan.–Feb. 2018, Jan. 2020, and Nov.–Dec. 2021.

pedia versions of Nov. 11, 2021.⁶⁵

Having completed his 18th year by June 1, 1937, Hasenjaeger was forced by law to do 6 months service according to the Reichsarbeitsdienstgesetz of June 26, 1935, immediately followed by 24 months service according to the Wehrgesetz of May 21, 1935. According to these laws, the lengths of these services are to be set by the “Führer and Reichskanzler”, i.e. by Adolf Hitler, who chose these times as given here.

So Hasenjaeger should have been drafted by the Nazi state by July 1937, unless something relevant for the Nazi regime spoke against it, such as activities relevant for warfare, which included *studying medicine*.

A possible source for the claim of a *voluntary* participation is a link newly added to the German Wikipedia on Jan. 10, 2020, pointing to (Schmeh, 2005), where we read: “Nach dem Abitur im Jahr 1936 meldete er sich zunächst freiwillig zum Arbeitsdienst, um anschließend studieren zu können. Da jedoch der Krieg dazwischen kam, wurde er zum Militärdienst eingezogen.” In (Schmeh, 2009, p. 343) we find an English version of these sentences: “After his graduation in 1936, he volunteered for labor service, planning to take up his studies afterwards. But the war intervened and he was drafted.”

Several aspects should be noted here: (1) The sentences are not very reliable as Hasenjaeger writes in his own CV (Hasenjaeger, 1997) that he graduated in 1937, which is perfectly in line with the German standard of entering school with 6 (i.e. in late summer 1925) and graduating 12 years later from 1937 on (before 1937: 13 years). (2) The war intervened not during Hasenjaeger’s labor service, but during his later military service. (3) In (Schmeh, 2009, p. 343) we read: “The author met him in 2005, a year before his death.” Therefore, the German sentence in (Schmeh, 2005) is closer to the communication of Klaus Schmeh with Hasenjaeger in 2005 and therefore more reliable. (4) This is relevant because the German version actually says something different: Not the labor service was voluntary, but his “Meldung” (“answer” or “notice”). Thus, after passing his obligatory muster (probably in late 1936), Hasenjaeger probably answered that he would prefer an early start of his labor service because he wanted to study mathematics (but not medicine!) after his labor and military services *as early as possible without any interruption of his studies*.

Acknowledgments

We would like to thank Beate Becker (née Hasenjaeger), Ludwig Bernays, Egon Börger, Rainer Glaschick, Norbert Hungerbühler, Ralf Krömer, Paolo Mancosu, Klaus Schmeh, and Wilfried Sieg for their most helpful comments, and Norbert Hungerbühler and Frieder Stolzenburg for their kind support.

⁶⁵Based on several similar corrections of mathematical errors, it really seems that any nonsense on Wikipedia reoccurs there again and again and that any effort to correct it turns out to be in vain in the end.

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